Superconducting Nanowire Based Devices

Gil Bachar
Superconducting Nanowire Based Devices

RESEARCH THESIS

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Gil Bachar

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Abstract

We investigate devices integrating superconducting nanowires.

First, we embed a nanowire in a superconducting microwave resonator. We show that there is a region of operation in which self excited oscillations arise. We describe the fabrication of the devices from a high temperature superconductor (YBCO). A simple theoretical model, introduced in an earlier work, is used to explain the phenomena. We investigate this model theoretically and make new predictions, which are then demonstrated experimentally. We apply the self-modulation phenomenon to the amplification of electron spin resonance signals.

Next, we turn to investigate devices in which the nanowire is used for the detection of single photons. We introduce a novel configuration in which the nanowire is fabricated on the tip of an optical fiber and inside an optical cavity. We investigate the device theoretically, and show that it can be used to achieve high detection efficiency at high rates. We then discuss the fabrication steps and provide experimental results for single-photon detection.
# Glossary

## Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AC</td>
<td>Alternate current</td>
</tr>
<tr>
<td>AS</td>
<td>Astable</td>
</tr>
<tr>
<td>BiS</td>
<td>Bistable</td>
</tr>
<tr>
<td>CW</td>
<td>Constant wave</td>
</tr>
<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>DI</td>
<td>Deionized</td>
</tr>
<tr>
<td>EBL</td>
<td>Electron beam lithography</td>
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<tr>
<td>ECR</td>
<td>Electron cyclotron resonance</td>
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<tr>
<td>ESR</td>
<td>Electron spin resonance</td>
</tr>
<tr>
<td>FBG</td>
<td>Fiber Bragg grating</td>
</tr>
<tr>
<td>FC-UPC</td>
<td>Fiber connector, ultra polished contact</td>
</tr>
<tr>
<td>FIB</td>
<td>Focused ion beam</td>
</tr>
<tr>
<td>LC</td>
<td>Limit cycle</td>
</tr>
<tr>
<td>LIA</td>
<td>Lock-in amplifier</td>
</tr>
<tr>
<td>MS</td>
<td>Monostable</td>
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<tr>
<td>MW</td>
<td>Microwave</td>
</tr>
<tr>
<td>NA</td>
<td>Network analyzer</td>
</tr>
<tr>
<td>NC</td>
<td>Normal conducting</td>
</tr>
<tr>
<td>PMMA</td>
<td>Poly-methyl-meth-acrylate</td>
</tr>
<tr>
<td>RIE</td>
<td>Reactive ion etching</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>SA</td>
<td>Spectrum analyzer</td>
</tr>
<tr>
<td>SC</td>
<td>Superconducting</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning electron microscopy</td>
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<tr>
<td>SM</td>
<td>Self-modulation</td>
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<tr>
<td>SMF</td>
<td>Single mode fiber</td>
</tr>
<tr>
<td>SNSPD</td>
<td>Superconducting nanowire single-photon detector</td>
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</table>
List of Symbols and abbreviation

\( \beta \)  Wave propagation constant
\( \gamma \)  Damping factor
\( \gamma_1 \)  Coupling constant between the resonator and the feedline
\( \gamma_2 \)  Unloaded damping rate of the resonance
\( \varepsilon_r \)  Relative dielectric constant
\( \varepsilon_{\text{eff}} \)  Effective dielectric constant
\( \zeta \)  Filling factor
\( \eta \)  Efficiency
\( \eta_C \)  Detector to light coupling efficiency
\( \eta_A \)  Light absorption efficiency
\( \eta_P \)  Pulse creation efficiency
\( \Lambda \)  Waveguide length
\( \lambda \)  Light wavelength
\( \mu_0 \)  Vacuum permeability
\( \rho \)  Specific resistance
\( \tau \)  Event time constant
\( \chi \)  Magnetic susceptibility
\( (\omega), f \)  (Angular) Frequency
\( (\omega_0), f_0 \)  (Angular) Resonance frequency
\( (\omega_p), f_p \)  (Angular) Pump frequency
\( (\omega_{\text{SM}}), f_{\text{SM}} \)  (Angular) Self-modulation frequency
\( \omega_1 \)  Angular Larmor frequency
1D  One-dimensional
\( |S_{11}| \)  Power reflection coefficient
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<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>A, $\langle A \rangle$</td>
<td>Electromagnetic mode (envelope) amplitude</td>
</tr>
<tr>
<td>$a^\text{in}, \langle a^\text{in} \rangle$</td>
<td>Incident injected power (envelope)</td>
</tr>
<tr>
<td>$B_z, \langle b_z \rangle$</td>
<td>Z-direction DC (AC) magnetic field</td>
</tr>
<tr>
<td>$B_z^{\text{ESR}}$</td>
<td>Magnetic field for electron spin resonance</td>
</tr>
<tr>
<td>$C$</td>
<td>Thermal heat capacity</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light in vacuum</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter</td>
</tr>
<tr>
<td>$E$</td>
<td>Electric field</td>
</tr>
<tr>
<td>$E_s(E_n)$</td>
<td>Electromagnetic mode envelope amplitude SC (NC) limiter</td>
</tr>
<tr>
<td>$(\hbar, h)$</td>
<td>(Reduced) Planck constant</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Wave number</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$H$</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>$I, \langle j \rangle$</td>
<td>Electrical current (density)</td>
</tr>
<tr>
<td>$I_c, \langle j_c \rangle$</td>
<td>Critical current (density)</td>
</tr>
<tr>
<td>$I_B$</td>
<td>Bias current</td>
</tr>
<tr>
<td>$I_{\text{in}}$</td>
<td>Input current</td>
</tr>
<tr>
<td>I-V (V-I)</td>
<td>Current to voltage (Voltage to current) relation</td>
</tr>
<tr>
<td>$L_k$</td>
<td>Kinetic inductance</td>
</tr>
<tr>
<td>$M$</td>
<td>Transfer matrix</td>
</tr>
<tr>
<td>$n$</td>
<td>Refraction index</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Critical temperature</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Bath temperature</td>
</tr>
<tr>
<td>$T_s(T_n)$</td>
<td>Steady-state super (normal) temperature</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Intermediate temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$P$</td>
<td>Power</td>
</tr>
<tr>
<td>$P_{\text{pump}}$</td>
<td>Pump power</td>
</tr>
<tr>
<td>$P_{\text{refl}}$</td>
<td>Reflected power</td>
</tr>
<tr>
<td>$Q$</td>
<td>Quality factor</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance</td>
</tr>
<tr>
<td>$R_B$</td>
<td>Microbridge resistance</td>
</tr>
<tr>
<td>$R_D$</td>
<td>Detector resistance</td>
</tr>
<tr>
<td>$R_L$</td>
<td>Load resistance</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>Characteristic impedance</td>
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Chapter 1

Introduction

Superconductivity is a phase transition that occurs in several materials when cooled to low temperatures. During the phase transition, several properties of the materials change. In this thesis, we employ devices with embedded narrow, thin, and long superconducting sections, which we refer as "superconducting nanowires" or "superconducting micro-bridges". Due to their dimensions, the nanowires have large resistivity in the normal state. On the other hand the nanowires have zero resistivity in the superconducting state. We set the devices to work at the edge of the phase transition, for example, by setting the current passing through them very close to the critical current. In this mode of operation, known as "bifurcation amplification", small external perturbations can be measured by an avalanche process: The small perturbation changes the local state of the nanowire to normal conductivity; the high resistivity of the nanowire causes current routing in the circuit; the routed current discharge energy stored in the entire device which is measured as an external output signal. As oppose to other bifurcation amplifiers, our devices also include self resetting mechanism, which allows the nanowires to cool down back to the superconducting state at the end of the detection event, and be ready for the next perturbation detection. In this thesis we utilize this mechanism and explore two types of devices.

1.1 Nonlinear RF Resonators

The nonlinear response of superconducting RF devices can be exploited for the benefit of a variety of applications such as noise squeezing [1], bifurcation amplification [2, 3, 4] and resonant readout of qubits [5]. In an early work done in our laboratory we reported an instability found in an NbN superconducting microbridge embedded in transmission line resonators [6, 7, 8]. In these experiments a monochromatic pump tone, having a frequency close to one of the resonance frequencies, is injected into the micro-bridge-embedded-resonator and the reflected power off the resonator is measured. We discovered that there is a certain zone in the pump frequency -
CHAPTER 1. INTRODUCTION

pump amplitude plane in which the resonator exhibits a limit-cycle (LC) response resulting in the self-sustained modulation (SM) of the reflected power. Moreover, to account for the experimental findings, we proposed a simple piecewise linear model which attributes the resonator’s nonlinear response to thermal instability occurring in the microbridge [9]. In spite of its simplicity, this model yields a rich variety of dynamical effects. The first part of this thesis further explores these phenomena, both experimentally and theoretically.

In chapter 2 we demonstrate the phenomena in a YBCO microwave RF resonator operating at temperatures of up to 77 K. We discuss the fabrication steps, the experimental setup, and the experimental data. We also apply the theoretical model to the experimental results of our new devices.

In chapter 3 we discuss the occurrence of intermittency, namely the coexistence of LC and a stationary solution, and noise-induced jumping between them. We further investigate the theoretical model, and employ a 1D map to find the possible LC solutions of the system and the conditions for the intermittency phenomena. We then compare the experimental results with the theoretical predictions.

In chapter 4 we utilize a microbridge-embedded microwave resonator for the detection of an electron spin resonance (ESR) signal in a magnetic sample. Again, experimental data are compared with the theoretical predictions. We show that the non-linear detection method can greatly enhance the ESR signal response to external excitation.

Finally, in chapter 5 we extend the theoretical model to a case in which several resonance modes interact with each other and with the microbridge. In this chapter, the full derivation of the two-mode case equations of motion is given and the system stability zones are analyzed. The experimental data are then compared to the theory.

1.2 On-Fiber SNSPD

Single-photon detectors and very sensitive photon detectors are needed in various fields of science, such as quantum communication [10, 11, 12], space communication [13], astronomy [14, 15], motion detection [16], molecule sequencing [17], artificial atom probing [18], and many others. The main figures of merit used to qualify a single photon detector are: the detection rate, the detection efficiency, the jitter, and the dark count rate. Superconducting nanowire single-photon detectors (SNSPD) [19, 20] are considered a promising technology for optical detection in the visible-to-near-infra-red band [21, 22]. The detection mechanism utilizes a fast avalanche process, in which a single photon is absorbed in the SNSPD and creates a resistive section in the nanowire. Current SNSPDs do show a fast response as well as negligible dark counts; however, they suffer from detection efficiency rates well below 100%.

The detection efficiency $\eta$, also referred to as the "quantum efficiency", is defined as the percentage of photons detected by the detector, out of those impinging on it.
1.2. ON-FIBER SNPSD

The detection efficiency can be calculated as $\eta = \eta_C \times \eta_A \times \eta_P$, where: the coupling efficiency $\eta_C$ is the percentage of photons that reach the detector out of those sent, the absorption efficiency $\eta_A$ is the percentage of photons absorbed in the detector out of those that reach it, and the pulse efficiency $\eta_P$ is the percentage of photons creating a pulse out of those absorbed.

Maximizing $\eta_C$ requires focusing the input light on a detector, which is typically defined as a square (or a circle) with sides (or diameter) that are a few micrometers long, operating inside a cryostat. Methods based on free space optics [20] or mechanical cryogenic positioning of an optical fiber in front of the sample [23, 24] require complicated and expensive instruments, and suffer from poor alignment stability. Alternatively, methods based on the fixed alignment of an optical fiber to the sample [25, 26] suffer from a fiber-center-to-detector-center misalignment of a few micrometers at least [27]. For these fixed alignment procedures, an increase in the detector's dimensions is inevitable in order to keep $\eta_C$ high. As the detector recovery time is linearly proportional to its area [28, 29], it is important to keep the detector as small as possible. For example, for a typical device, made of 100 nm wide 5 nm thick niobium-nitride wires folded to an area of $25 \mu m^2$, the recovery time is 2.5 ns [28, 29].

In order to maximize $\eta_A$, an optical cavity can be used. In prior art designs, the detector acts as one mirror and a second, metallic, mirror is added [30]. However these cavities have low finesse due to the fact that the detector is nearly transparent.

In chapter 6, we propose a novel system configuration in which the detector is fabricated on the tip of an optical fiber and inside an optical cavity. The on-fiber fabrication allows precise alignment of the detector to the core, where the light intensity is maximal, achieving high coupling efficiency, while keeping the device small and fast. The optical cavity is defined between two mirrors. The first mirror is a dielectric Bragg grating mirror, and the second mirror is metallic mirror. Control over the first mirror allows tuning the cavity to critical coupling, a mode of operation in which $\eta_A$ is close to unity. The proposed detector makes it possible to greatly increase in the detection efficiency. It is also simple to operate, small in dimensions and does not require complicated optical or mechanical equipment. Thus, our detector can fit many of the applications mentioned above.

Chapter 6 is organized as follows: First, we provide a theoretical model, both of the detection mechanism and of the optical cavity formation. Then we discuss the fabrication steps and the experimental setup. Finally, we present the experimental results.
Chapter 2

Nonlinear RF YBCO Resonators

In this chapter we present the phenomenon of self-modulation in YBCO resonators. We first describe the electromagnetic design considerations, the fabrication steps and the cryogenic setup. Then we discuss the experimental data from our devices: we analyze the microbridges working in a DC regime; the RF response of our resonator; and the nonlinear response of self-modulation (SM) due to the presence of the microbridge in the resonator. We show, for the fist time, the occurrence of SM phenomena at 77 K temperature. Finally we compare the experimental data to a theoretical model.

2.1 RF Resonator Design

We have designed stripline and coplanar waveguides microwave resonators [31, 32], that have a characteristic impedance of 50 Ω. The first three resonance frequencies were designed in the range of 2 GHz – 10 GHz. The resonators are capacitively weakly coupled to a feedline. In each resonator, a narrow section (the "microbridge") is defined with characteristic length of a few microns and width of 0.5 – 1 μm. This microbridge is responsible for the strong nonlinearity in the resonator’s response, as explained in chapter 2.7. All resonator parameters are summarized in table 2.1.

2.2 Fabrication Steps

The fabrication process starts with 0.5-mm-thick sapphire wafers coated with 150 nm of YBCO and 200 nm of gold cap layer. Gold contacts and alignment marks are patterned using standard photolithography and wet etching in a KI–I–H2O solution. The waveguides are patterned with electron beam lithography or photolithography. The YBCO layer is then etched either by 30 minutes of wet etching with phosphoric acid or 5 minutes of dry etching (argon ion milling). In the final step, the microbridges are patterned using a gallium focused ion beam (FIB) machine. In order to avoid charging problems during the FIB process, the whole wafer is first coated with 5 nm of
gold, which is afterwards removed using the KI–I–H$_2$O solution. Further fabrication details are given in the appendix 2.B

<table>
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<tr>
<th>num</th>
<th>W.G</th>
<th>$f_{0d}$[GHz]</th>
<th>$f_{0m}$[GHz]</th>
<th>Q factor ($\times 10^3$)</th>
<th>Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1.1</td>
<td>Stripline</td>
<td>2.27 × n</td>
<td>2.65, 5.26, 8.5</td>
<td>0.3, 0.7, 0.3</td>
<td>stripe</td>
</tr>
<tr>
<td>G2.3</td>
<td>Stripline</td>
<td>2.27 × n</td>
<td>2.86, 3.62, 8.37, 8.41, 8.47</td>
<td>14.7, 6.6, 1, 9, 30</td>
<td>meander</td>
</tr>
<tr>
<td>G3.1</td>
<td>Stripline</td>
<td>2.27 × n</td>
<td>2.72, 5.322, 7.09</td>
<td>3.5, 1, 0.688</td>
<td>meander</td>
</tr>
<tr>
<td>G3.2</td>
<td>Stripline</td>
<td>2.27 × n</td>
<td>2.9, 5.7, 8.6</td>
<td>34, 14, 5</td>
<td>meander</td>
</tr>
<tr>
<td>G2.2</td>
<td>Coplanar</td>
<td>3.07 × n</td>
<td>8.848, 8.859, 8.96</td>
<td>0.9, 1.2, 4.5</td>
<td>meander</td>
</tr>
<tr>
<td>G5.4</td>
<td>Coplanar</td>
<td>3 × n</td>
<td>8.848, 8.859, 8.96</td>
<td>0.9, 1.2, 4.5</td>
<td>meander</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters of various resonators measured. W.G is the waveguide type, $f_{0d}$ is the designed resonance frequency ($n$ is an integer), $f_{0m}$ is the measured resonance frequency, at 4.2 K, by $|S_{11}|$ measurement with −25 dBm input power. Only frequencies between 2 GHz and 10 GHz showing evidence of SM phenomena are listed (see section 2.5). Note that the measured resonance frequencies vary between different cooling cycles.
2.3. CRYOGENIC COOLING

![Image of cryogenic cooling devices](image.png)

Figure 2.2: (Color) (A) Copper box holding the sample, for electromagnetic noise reduction, (B) Sample containing 4 stripline and 2 coplanar resonators, (C) Sample containing 3 coplanar resonators.

2.3 Cryogenic Cooling

The devices are mounted into a copper box which helps reducing RF electromagnetic losses and decrease electromagnetic noise from the environment (see Fig. 2.2). The devices are cooled in a vacuum chamber filled with low-pressure helium exchange gas, which is immersed in liquid helium or liquid nitrogen coolant. The temperature inside the chamber can then be set and monitored using a heater (heating the chamber), a pump (pumping gas vapor and cooling the cryogenic liquid) and a thermometer.

2.4 DC-Biased YBCO Microbridges

In order to study the DC behavior of the YBCO micro bridges, we fabricated a series of test samples, each containing a microbridge connected via 4-probe contacts.

During the FIB etching process bridges suffer from heating and gallium contamination [33]. The effective width of the bridges is thus assumed to be smaller than the one measured using electron microscopy. We used a simple approximation for the bridges’ effective width, by assuming that the FIB creates damage at a constant penetration depth on the perimeter of the patterned area. We assumed that the penetration depth is dependent on the ion milling duration and current but not on the width of bridge, namely: \( w_{\text{b,eff}} = w_b - 2w_p \), where \( w_b \) is the width of the bridge and \( w_p \) is the assumed narrowing factor due to gallium poisoning [33]. The normal resistance...
of the bridge can then be evaluated, using the relation:

$$R_b = \frac{l_b}{d \times (w_b - 2w_p) \rho_n},$$  \hspace{1cm} (2.1)

where $R_b$ is the normal bridge resistance, $\rho_n$ is YBCO-specific resistance, $l_b$ is the bridge length and $d$ is the film thickness. From which we deduce $w_p \approx 110$ nm. In previous reports $w_p$ was found to be $30 - 50$ nm in Nb [34, 35]; our results have similar order of magnitude, and the difference can be attributed to the change of material, milling time and current.

The devices were then mounted into the cryostat system (described in the "Measurement Setup" section), and cooled with liquid nitrogen (which was cooled down to 74 K by pumping the vapor above the liquid). The exact temperature of the devices could be set using a temperature controller, and a heater. Figure 2.5 shows temperature-dependent I-V curves taken at different temperatures. Superconducting transition is above the temperature of 77 K. The hysteresis in the data, as well as the jumps, are similar to results obtained by Mikheenko et al. [36] with similar YBCO bridges.

### 2.5 Resonance Frequencies of the System

Our resonators are designed as half-lambda transmission line resonators, where the bridge is modeled as shunt impedance at some point. For such a resonator, the
2.5. RESONANCE FREQUENCIES OF THE SYSTEM

Figure 2.4: YBCO bridges’ gallium poisoning. \( w_b \) vs. \( 1/R \) is plotted. For all bridges \( l_b = 10\mu m \) and \( d = 150\,nm \). The red circles show experimental data, and the blue solid line is a fit to \( \rho_n = 4.515\mu \Omega \,m \) and \( p = 108.5\,nm \).

Resonance (angular) frequencies, \( (f_0) \), \( \omega_0 \) can be approximated, when the impedance of the bridge is small, by [31]:

\[
\omega_0 = 2\pi f_0 = 2\pi \times n \times \frac{c}{\sqrt{\varepsilon_{\text{eff}}}} \frac{2}{\Lambda},
\]

where \( \Lambda \) is the waveguide length, \( c \) is the speed of light, \( \varepsilon_{\text{eff}} \) is the effective permittivity of the material and \( n \) is a positive integer. This formula is a good approximation of the resonance frequencies provided that the shunt impedance is small. A full derivation of the resonance frequencies for a case of a finite shunt impedance is given in appendix 2.A. In addition to the transmission line resonators, the copper box, which is a finite rectangular waveguide, also has internal resonance frequencies. The first few resonance frequencies of the copper box can be calculated by [31]

\[
\omega_{\text{box}} = \frac{\pi c}{\sqrt{\varepsilon_{\text{eff}}}} \left[ \left( \frac{n}{a} \right)^2 + \left( \frac{m}{b} \right)^2 \right]^{1/2},
\]

where \( a, b \) are the 2 (out of 3) longer dimensions the copper box (30 and 34 mm in our case) and \( n, m \) are positive integers. In order to identify the resonance frequencies, the resonator reflection coefficient as a function of frequency is measured using a network analyzer (NA). For each frequency, the NA transmits a single coherent tone and measures the amplitude of the reflected signal. This measurement is referred hereafter as a \( |S_{11}| \) measurement. Figure 2.6 shows a typical \( |S_{11}| \) measurement from one of our resonators. A resonance frequency is identified as a deep in the reflection coefficient.
Figure 2.5: (color) I-V curves of a YBCO microbridge at different temperatures. The solid lines are measurements while increasing the current from 0 mA to 2.5 mA and the dashed lines are measurements while decreasing the current back to zero. A clear hysteresis can be seen at lower temperatures. The hysteresis can be explained by the fact that the normal current generates local Joule heating of the microbridge, which results in reduced $I_c$.

At resonance, most of the energy is dissipated in the resonator rather than being reflected back. The resonator’s quality (Q) factor is defined as the ratio between the frequency of the resonator and its damping rate. In Fig. 2.6 the resonator’s resonance frequencies are marked with red arrows, whereas the resonance frequencies of the box are marked with black arrows. The resonator’s resonance frequencies show strong dependence on the input power; when the input power is increased over a given threshold, the $|S_{11}|$ line shape broadens and become more shallow, implying that the Q-factor decreases. This behavior is explained by a theoretical model which is developed in section 2.7.

## 2.6 Self-Sustained Modulation in YBCO Resonators

The experimental setup used for the investigation of self-sustained modulation (SM) can be seen in Fig. 2.3. A single coherent tone, with frequency $f_r$ and power $P_r$,
is injected into the resonator. The reflected signal is divided and measured both by a spectrum analyzer in the frequency domain, and by an oscilloscope, that monitors the slow envelope of the signal in the time domain.

Figure 2.7 demonstrates the phenomenon of SM. Panel A shows a spectral analysis of the reflected signal at a point of operation where SM occurs. In this case, the reflected signal has low frequency modulation over the fast frequency of \( f_p \). The slow envelope is periodic with time \( \tau_{SM} \). The SM frequency, \( f_{SM} \) is defined as \( f_{SM} = 1/\tau_{SM} \). In panel A, the center frequency is \( f_p \), whereas the side bands around it are at frequencies \( f_p \pm k \times f_{SM} \), where \( k \) is an integer. SM was observed in YBCO resonators up to a temperature of 77 K.

Panel B shows the spectrum of the reflected signal off the resonator, while keeping input frequency constant and changing the input power. Below a lower threshold (−1.7 dBm in this case) and above an upper threshold (−0.2 dBm) the behavior of the device is linear, namely the only reflected tone has the same frequency as the input pump, \( f_p \). Between the lower and upper power thresholds, the device shows
Figure 2.7: (Color) Self sustained modulation in G3.1 YBCO resonator. (A) Spectral analysis of the reflection from the sample at $f_p = 8.43$ GHz and $P_p = -1$ dBm. The SM phenomena is seen with at frequency $f_{SM} = 30$ MHz. (B) Spectral analysis of a scan over input power, while input frequency is held constant at $f_p = 8.43$ GHz. The colors represent the power in each spectral component, $P_{ref}$. (C) Experimental data of the SM diagram. (D) Numerical simulation of the SM diagram. For panels (C) and (D) the colors represent the measured and expected $f_{SM}$, respectively.

A strong nonlinear SM response. Similar thresholds also exists when scanning the frequency domain; The SM phenomenon occurs only in a frequency range close to the resonance frequency of the resonator.

We define the SM diagram is a 3D plot of $\omega_{SM}$ vs. $\omega_p$ and $P_p$. The SM diagram is only defined in points were SM exists. Figure 2.7-C shows an experimental SM diagram obtained around the third resonance frequency of resonator G3.1, where the color map indicates $\omega_{SM}$. Panel D shows a numerical simulation that generates a similar diagram for the same resonance mode, using the model to be discussed in section 2.7. This simulation accounts for the shape of the region and the SM frequencies.
2.7 SM Theoretical Model

Equations of Motion

The dynamics of our system can be captured by two coupled equations of motion, which are briefly described herein (see Ref. [9] for a detailed derivation). Consider a resonator driven by a weakly coupled feedline carrying an incident coherent tone \( a^{\text{in}} = a^{\text{in}} e^{-i\omega_0 t} \), where \( a^{\text{in}} \) is a constant complex amplitude and \( \omega_0 \) is the driving angular frequency. The mode amplitude inside the resonator can be written as \( A = A(t) e^{-i\omega_0 t} \), where \( A(t) \) is a complex amplitude, which is assumed to vary slowly on a time scale of \( 1/\omega_0 \). In this approximation, the equation of motion of \( A \) reads

\[
\frac{dA}{dt} = [i(\omega_0 - \omega_0) - \gamma] A - i\sqrt{2\gamma} a^{\text{in}} + c^{\text{in}},
\]

where \( \omega_0 \) is the angular resonance frequency and \( \gamma = \gamma_1 + \gamma_2 \), where \( \gamma_1 \) is the coupling coefficient between the resonator and the feedline and \( \gamma_2 \) is the damping rate of the mode. The term \( c^{\text{in}} \) represents an input Gaussian noise, whose time autocorrelation function is given by

\[
\langle c^{\text{in}}(t)c^{\text{in}}(t') \rangle = G\omega_0 \delta(t - t'),
\]

where the constant \( G \) can be expressed in terms of the effective noise temperature \( T_{\text{eff}} \) as

\[
G = \frac{2\gamma k_B T_{\text{eff}}}{\omega_0 \hbar \omega_0}.
\]

Note that in our experiment, in addition to thermal contribution, a phase noise of the signal source also makes a significant contribution to the total noise, and consequently the effective temperature of the noise \( T_{\text{eff}} \) can be higher than coolant temperature and even higher than room temperature. The effective noise temperature \( T_{\text{eff}} \) is determined by numerically fitting to experimental data.

We consider the case where non-linearity is generated by a local hot-spot in the resonator, i.e. the microbridge. The microbridge is assumed to be sufficiently small, so that its temperature \( T \) may be considered homogeneous. The temperature of all other parts of the resonator is assumed to equal that of the coolant \( T_0 \). The heat balance equation for the microbridge reads

\[
C \frac{dT}{dt} = 2\hbar \omega_0 \gamma_2 \alpha |A|^2 - H(T - T_0),
\]

where \( C \) is the thermal heat capacity, \( \alpha \) is the portion of the heating power applied to the microbridge relative to the total power dissipated in the resonator (0 \( \leq \alpha \leq 1 \)), \( H \) is the heat transfer coefficient.

The coupling of Eqs. (2.4) and (2.7) originates in the dependence of the parameters of the driven mode \( \omega_0, \gamma_1, \gamma_2 \) and \( \alpha \) on the microbridge’s resistance and inductance, which in turn depend on its temperature. We assume the simplest case, where this
dependence is a step function that occurs at the critical temperature $T_c \simeq 10$ K of the superconductor, namely

$$\omega_0, \gamma_1, \gamma_2, \alpha = \begin{cases} \omega_0 s, \gamma_1 s, \gamma_2 s, \alpha_s & T < T_c \\ \omega_0 n, \gamma_1 n, \gamma_2 n, \alpha_n & T > T_c \end{cases}. \tag{2.8}$$

The assumption that the parameters characterizing the resonator have a step function dependence on temperature greatly simplifies the problem at hand since this assumption yields piecewise linear dynamics. In reality, however, the transition has a finite width (see our previous work [37]). In order to investigate the dependence on the transition width we have substituted the step function dependence by a hyperbolic tangent dependence to model a smooth transition. We found, however, that the results remain almost unchanged provided that the temperature peak-to-peak amplitude of self oscillations is much larger than the transition width.

**Steady-State Solutions**

Solutions of steady-state response to a monochromatic excitation are found by seeking stationary solutions to Eqs. (2.4) and (2.7) for the noiseless case $e^{in} = 0$. Steady state can be achieved only if the two following conditions are fulfilled simultaneously:

$$\begin{align*}
\frac{dA}{dt} &= 0, \tag{2.9a} \\
\frac{dT}{dt} &= 0. \tag{2.9b}
\end{align*}$$

The system may have, in general, up to two locally-stable steady-states $\{A_s, T_s\}$ and $\{A_n, T_n\}$, corresponding respectively to the SC and NC phases of the microbridge. The stability of each of these phases depend on the corresponding steady state values of the envelope and the temperature. Replacing Eq. (2.9) in Eqs. (2.4) and (2.7) for both SC and NC phases

$$\begin{align*}
A_s &= \frac{i \sqrt{2} \gamma_1 a^{in}}{i (\omega_p - \omega_0 - \gamma)}; \\
T_s &= \frac{2 \hbar \omega_0 \gamma_2 \alpha_s |A_s|^2}{H} + T_0, \tag{2.10b}
\end{align*}$$

and

$$\begin{align*}
A_n &= \frac{i \sqrt{2} \gamma_1 a^{in}}{i (\omega_p - \omega_0 - \gamma)}; \\
T_n &= \frac{2 \hbar \omega_0 \gamma_2 \alpha_n |A_n|^2}{H} + T_0. \tag{2.11b}
\end{align*}$$
2.A. STANDING WAVE RELATIONS

An SC steady state exists only if $T_s < T_c$, which leads to the condition $|A_s|^2 < E_s$ where

$$E_s = \frac{H (T_c - T_0)}{2 \hbar \omega_0 \gamma_{2s} \alpha_s}.$$  \hspace{1cm} (2.12)

Similarly, a NC steady state exists only if $T_n > T_c$, meaning $|A_n|^2 > E_n$ where

$$E_n = \frac{H (T_c - T_0)}{2 \hbar \omega_0 \gamma_{2n} \alpha_n}.$$  \hspace{1cm} (2.13)

Consequently, four stability zones can be identified in the plane of pump power $P_p \propto |a_0|^2$ - pump frequency $\omega_p$ (see Fig. 3.1) [9]. Two are mono-stable (MS) zones (MS(SC) and MS(NC)), where either the SC or the NC phases is locally stable, respectively. Another is a bistable zone (BS), where both phases are locally stable. The third is an astable zone (aS), where none of the phases are locally stable.

2.A Standing Wave Relations

Consider a wave propagating along a transmission line resonator, with an embedded lumped impedance element. The transmission line is assumed to have impedance $Z_0$. We assume the resonator length is $\Lambda_1 + \Lambda_2$, where the beginning of the resonator (coupling to a feedline) is at $z = -\Lambda_1$, the lumped impedance element is at $z = 0$ and the end of the resonator is at $z = \Lambda_2$. Using standing wave analysis the voltage drop along the resonator is given by:

$$V(z) = \begin{cases} V_A \cos(\beta z) + V_B \sin(\beta z) & -\Lambda_1 < z < 0 \\ V_C \cos(\beta z) + V_D \sin(\beta z) & 0 < z < \Lambda_2 \end{cases}.$$ \hspace{1cm} (2.14)

The current is given by $I(z) = (i/\beta Z_0)(dV/dz)$ or

$$I(z) = \frac{i}{Z_0} \begin{cases} -V_A \sin(\beta z) + V_B \cos(\beta z) & -\Lambda_1 < z < 0 \\ -V_C \sin(\beta z) + V_D \cos(\beta z) & 0 < z < \Lambda_2 \end{cases}.$$ \hspace{1cm} (2.15)

The boundary conditions are: $I(-\Lambda_1) = I(\Lambda_2) = 0$, $V(0^+) - V(0^-) = ZI(0)$, $I(0^+) = I(0^-)$. From these conditions we conclude that:

$$V_A = -V_B \cot(\beta \Lambda_1),$$
$$V_C = V_D \cot(\beta \Lambda_2),$$
$$V_B = V_D,$$ \hspace{1cm} (2.16)
$$V_C - V_A = i \frac{Z}{Z_0} V_B.$$

i.e the resonance condition for $\beta$ is:

$$\cot(\beta \Lambda_2) + \cot(\beta \Lambda_1) = i \frac{Z}{Z_0}.$$ \hspace{1cm} (2.17)
CHAPTER 2. NONLINEAR RF YBCO RESONATORS

Using the relation

$$\beta = \sqrt{\varepsilon_{\text{eff}}} \frac{\omega_0 + i\gamma_2}{c},$$

we can calculate \(\omega_0\), the resonance frequency of the resonator, and \(\gamma_2\), the damping rate of the mode. Note that for the case of \(Z \to 0\), we get that \(\beta \to \frac{k\pi}{\Lambda}\), meaning that \(\omega_0 \to k\pi \frac{c}{\sqrt{\varepsilon_{\text{eff}}}}\).

2.B Step-by-Step Fabrication Recipe

The following procedure was used for the fabrication of the YBCO resonator.

1. Obtain base samples (in our case the samples were purchased from THEVA GmbH, Germany). The basic sample structure is:
   a) 0.5-mm-thick, 30-mm-wide, 34-mm-long sapphire crystal (Al\(_2\)O\(_3\)), C-axis cut.
   b) 150-nm-thick YBCO layer.
   c) 200-nm-thick gold layer, deposited in-situ (without breaking the vacuum in the chamber), on top of the YBCO layer.

2. Thoroughly clean the wafer in solvents: acetone, methanol, and isopropanol. Clean the wafer for at least 5 minutes per solvent and then spin dry to remove access isopropanol.

3. Pattern alignment marks and contacts using photolithography. In our case, the mask for this step was ordered from external suppliers. Alignment crosses should comply with the requirements of electron beam lithography (EBL) or photolithography to follow. Contact pads should comply with launchers provided in the packaging box.
   a) Dry the sample on a hot plate at 110°C for 10 min.
   b) Deposit 4533 or 1818 photoresist, by spinning at 5000 rpm for 60 s.
   c) Bake the sample on a hot plate 110°C for 1 min.
   d) Expose the sample to UV light for 5 – 15 s. The exact time is dependent on the microscope and the resist age.
   e) Develop the photoresist in a TMAH solution (diluted in DI water at a 1:10 ratio) for 40 – 60 s.
   f) Dry the sample by spinning it at 2000 rpm for 1 min.

4. Etch the gold layer:
2.B. **STEP-BY-STEP FABRICATION RECIPE**

a) Mix KI - I₂ - H₂O at a ratio of 1:4:40. The solution can be kept in a sealed glass container for a few months, and can be reused at least ten times (for a quantity of about 100 cc).

b) Etch gold for about 10 s. The exact etching time is dependent on the age of the solution.

c) Rinse in DI water for 1 min. As YBCO is sensitive to water, it is critical that this step will not be significantly longer.

d) Rinse in Isopropanol for 1 min.

e) Spin the sample dry to remove excess isopropanol.

f) Further dilution of the etching solution in DI water may increase etching time and decrease the risk of undercuts, especially in the case of small features.

5. Remove the resist using N-methyl-2-pyrrolidone (NMP) for 1 hour. Clean by repeating step 3.

6. Lithograph the resonators using one of the following methods:

   a) Photolithography: this is a faster method. However, it requires having a mask of the resonator, and is thus less flexible in determining the resonator parameters for each sample (length and coupling gap). The process details are similar to step 3 in this chapter.

   b) Electron Beam Lithography (EBL): EBL is a slow process, since the electron beam can only write on very small area at a time (a circle with the diameter of the beam). However, the process is flexible and allows changing of the pattern between writing cycles. The process includes the following steps:

      i. Deposit Ma-N 2405 negative e-beam resist, by spinning at 5000 rpm for 60 s.

      ii. Bake the sample on a hot plate at 110°C for 5 min.

      iii. Expose the resist to the e-beam. Parameters are machine-dependent, and should be calibrated for different substrates and resist ages.

      iv. Develop the resist using MA-D 532 developer for 1 min.

7. Etch the YBCO layer. Two methods can be used:

   a) Wet etching: this is a simpler process: it requires less calibration, and ensures that the resist (photo or e-beam) will not be burned. However, patterns tend to be less accurate due to undercuts.
CHAPTER 2. NONLINEAR RF YBCO RESONATORS

i. Dilute phosphoric acid in DI water in ratio of 2% - 5%. The solution can be kept in a sealed plastic or teflon container for a few months, and reused at least ten times (for quantity of about 100 cc).

ii. Etch the YBCO protected by the resist mask from previous steps for about 5 min.

iii. Rinse in DI water for 1 min.

iv. Spin it dry to remove excess water.

v. Examine the sample under an optical microscope. Make sure that the unwanted YBCO is being etched (black areas become clearer), and that there is no significant undercut in the protected areas.

vi. Repeat steps 7(a)iii-7(a)vi for a total etching time of about 30 min, and until all unwanted YBCO is removed. If undercuts are created, phosphoric acid should be further diluted.

b) Dry etching: dry etching is prominent for less undercuts and smooth boarder lines. However, the machines’ vacuum chamber loading time, and the risk of burning the resist makes the process less attractive. For ion milling, 4533 resist is the best option.

i. Attach the sample to the ion milling stage in such a way as to ensure good thermal conductance, in order to remove excess heat during the milling process.

ii. Pump the ion chamber to a residual pressure not exceeding 2.5 μTorr.

iii. Cool the stage to 4°C. using water or other cold liquid.

iv. Fill the chamber with argon gas at residual pressure of 200 μTorr.

v. Run the ion milling at 250 V voltage, 110 mA in cycles of 1 min for a total etching time of 5 min. Wait at least 3 minutes between cycles. The total number of cycles is almost linearly dependent on YBCO thickness. 4533 resist can endure up to 1 hour of direct etching in these parameters.

vi. Pump the chamber to the base pressure, and wait at least 30 min before opening the vacuum chamber.

8. Remove the resist using N-methyl-2-pyrrolidone (NMP) for 1 hour. Clean by repeating step 3.

9. Deposit 5 nm of gold layer using thermal evaporation. This layer is used to avoid charging in the FIB lithography to follow. It can be avoided in case the substrate is a better conductor than sapphire (extrinsic silicone, for example).

10. Etch microbridges using focused a ion beam (FIB) machine. machine was operated at 9.7 μA current. See section 2.4 for limitation of FIB usage.
2.B. STEP-BY-STEP FABRICATION RECIPE

11. Repeat step 4 to remove gold layer.
Chapter 3

Intermittency in Self-Modulation Phenomena

3.1 Introduction

In this chapter we study both theoretically and experimentally noise-induced transitions between different metastable responses in our non-linear superconducting resonators. We employ a 1D map to identify the possible LC solutions of the system and to find conditions for the occurrence of the intermittency. Experimentally, we present measurements showing both, intermittency between an LC and a steady state, and intermittency between different LCs. A comparison between the experimental results and theory yields a partial agreement.

Intermittency is a phenomenon in which a system response remains steady for periods of time (the laminar phase) which are interrupted by irregular spurts of relatively large amplitude dynamics (the turbulent phase). It arises in certain deterministic systems that are near a bifurcation in which a steady response is destabilized or destroyed [38]. This phenomenon also occurs in noisy systems in which the laminar response has a weak point in its local basin of attraction and is randomly bumped across the basin threshold, and then ultimately reinjected back to the laminar state, and the process repeats. This latter type of bursting behavior, which is relevant to the present system, is observed to occur in many other systems, including Rayleigh-Bénard convections [39], acoustic instabilities [40], turbulent boundary layers [41], semiconducting lasers [42], blinking quantum dots [43], sensory neurons [44], cardiac tissues [45], micro and nano mechanical systems [46, 47, 48] and Josephson junctions [49]. The presence and level of noise has a significant effect on all such systems, since perturbations affect the triggering of the system out of the laminar phase [50, 51]. The mean duration times of the laminar phase for a certain class of these systems are scaled in a manner that depends on the bifurcation parameter and the noise level [52, 53]. A special feature of the present system is that it exhibits a very sharp transition between two types of
operating states, namely, normal conducting (NC) and superconducting (SC), which is modeled by equations with discontinuous characteristics. While the deterministic behavior of such non-smooth systems (at least of low order) is generally well understood, including local and global bifurcations [54], the effects of noise on such systems has not been considered.

3.2 Theory

Following the introduction of the equations of motion of our system, in section 2.7, we now wish to follow the time evolution of Eqs. (2.4) and (2.7), in order to find limit cycle (LC) solutions in the different stability zones (see Fig. 3.1). The task of finding LC solutions to these two equations can be greatly simplified by exploiting the fact that, in our devices, the dynamics of the mode envelop amplitude \( A \) [Eq. (2.4)] can be considered to be slow in comparison with the dynamics of temperature \( T \) [Eq. (2.7)]. This relationship can be expressed as

\[
\gamma \ll H/C,
\]

(3.1)

where \( \gamma = \gamma_1 + \gamma_2 \), \( \gamma_1 \) is the coupling coefficient between the resonator and the feedline, \( \gamma_2 \) is the damping rate of the mode, \( C \) is the thermal heat capacity and \( H \) is the heat transfer coefficient. In this limit one finds by employing an adiabatic approximation [9] that the temperature \( T \) remains close to the instantaneous value given by

\[
T_i = T_0 + \frac{2\hbar \omega_0 \gamma_2 a}{H} |A|^2,
\]

(3.2)

for most of the time except for relatively short time intervals (on the order of \( C/H \)) immediately following each switching event between the SC and the NC phases. Consequently, as can be seen from the example trajectories shown in Fig. 3.2 (A-1), transitions from the SC to the NC phase occur near the circle \( |A|^2 = E_s \), whereas transitions from the NC to the SC phase occur near the circle \( |A|^2 = E_n \), where \( E_n \) and \( E_n \) are the separatrices between the stable and unstable zones defined in Eqs. (2.12) and (2.13).

The important features of the system’s dynamics can be captured by constructing a 1D map [55]. Consider the case where \( E_n < E_s \) and the amplitude \( A \) lies initially on the circle \( |A|^2 = E_n \), namely \( A = \sqrt{E_n} e^{2\pi i x} \) where \( x \in [0, 1] \). Furthermore, assume that initially the system is in the SC phase, namely, \( T < T_c \) and consequently \( A \) is attracted towards the point \( A_n \). The 1D map \( D(x) \) is obtained by tracking the time system’s evolution for the noiseless case (\( e^{in} = 0 \)) until the next time it returns to the circle \( |A|^2 = E_n \) to a point \( A = \sqrt{E_n} e^{2\pi i D(x)} \) where \( D(x) \in [0, 1] \). In the adiabatic limit this can be done using Eq. (2.4) only [without explicitly referring to Eq. (2.7)] since switching to the NC phase in this case occurs when the trajectory intersects with the circle \( |A|^2 = E_n \). Note that in the aS zone of operation all points on the circle
3.2. THEORY

Figure 3.1: Stability zones in the $\omega_p - P_p$ plane: SC monostable (MS(SC)), NC monostable (MS(NC)), bistable (BiS), and astable (aS) (gray colored) zones. The region where a stable LC exists is marked with a dashed line. The inset shows the three operating points (A, B and C) at which the measurements and theoretical analysis shown in Fig. 3.2 and 3.3 are done. The following parameters were used in the numerical simulation: $\omega_{0s}/2\pi = 3.49$GHz, $\gamma_{1s} = 1.14e - 3\omega_{0s}$, $\gamma_{2s} = 2.74e - 3\omega_{0s}$, $\omega_{0n}/\omega_{0s} = 1.017\omega_{0s}$, $\gamma_{1n} = 1.14 \times 10^{-2}\omega_{0s}$ and $\gamma_{2n} = 2.74 \times 10^{-2}\omega_{0s}$, $C = 15.4$fJ/K, $H/C = 0.211\omega_{0s}$, $T_{eff} = 700$K. $\omega_p = 0.997\omega_{0s}$, $P_p = -25.34$ dBm (A), $-25.78$ dBm (B), $-26.75$ dBm (C).

$|A|^2 = E_n$ return back to it after a finite time. However, this is not necessarily the case in the other stability zones. Therefore, we restrict the definition of the 1D map $D(x)$ only to points on the circle $|A|^2 = E_n$ that eventually return to it. Other points have a trajectory that ends at a steady state (NC or SC), and thus their existence shows that one of these states is stable. Those points do not appear in the 1-D map.

Any fixed point of the 1D map, namely a point for which $D(x_0) = x_0$, represents an LC of the system. The LC is locally stable provided that $|dD/dx|_{x=x_0} < 1$ [55]. We have scanned the $\omega_p - P_p$ plane and, using a 1-D map for each working point, we were able to determine the region where an LC solution exists, which is marked with a dashed line in Fig. 3.1. Note that this region extends beyond the aS region due to the possibility of intermittency of a steady state solution and an LC one.
Figure 3.2: (Color) Non linear resonator dynamics. Subplots A, B and C correspond to the three operating points A, B and C respectively which are marked in the inset of Fig. 3.1. In subplot (A) only an LC is locally stable, in subplot (B) intermittency between an LC and an SC steady state occurs, whereas only an SC steady state is locally stable in subplot (C). In panels (A-1), (B-1) and (C-1), which show the time evolution in A plane, a plus sign labels $A^s$ and a cross sign labels $A_n$. These points are shown for reference and correspond to fixed points of the dynamics only when they exist in their respective domains, as defined in the text. Trajectories that return to the inner circle $|A|^2 = E_n$ are colored in blue (dark), and trajectories that end at $A^s$ are colored in yellow (gray). Panels (A-2), (B-2) and (C-2) show the corresponding 1D maps.
3.3. EXPERIMENTAL SETUP

Figure 3.3: Numerical [panels (A-3), (B-3) and (C-3)] vs. experimental [panels (A-4), (B-4) and (C-4)] time traces for the three operating points A, B and C respectively.

Figure 3.4: (A) Measurement setup. (B) Schematic layout of the device. (C) Optical microscope image of the straight-shaped microbridge.

3.3 Experimental Setup

The present experiments are performed using the setup depicted in Fig. 3.4(A). The resonator is stimulated using a monochromatic pump tone that has an angular frequency $\omega_p$ and power $P_p$. The power reflected off the resonator is amplified at room temperature and measured using both a spectrum analyzer in the frequency domain and an oscilloscope, tracking the reflected power envelope, in the time domain. All measurements are carried out while the device is fully immersed in liquid helium. A simplified circuit layout of the device is illustrated in Fig. 3.4(B). The resonator used
in these experiments is formed as a stripline ring made of niobium nitride (NbN) deposited on a sapphire wafer [37, 56], and having a characteristic impedance of 50 Ω. A feedline, which is weakly coupled to the resonator, is employed for delivering the input and output signals. A microbridge is monolithically integrated into the structure of the ring [57]. Further design considerations, fabrication details as well as normal modes calculation can be found in work by Segev et al. [37, 6].

3.4 Experimental and Numerical Data

Figure 3.2 shows noiseless the behavior of the resonator for the three operating points A, B and C, which lie near the border between the aS and the MS(SC) regions, and are marked in the inset of Fig. 3.1. Figure 3.3 shows a comparison between experimental data and numerical simulations for these operating points. The sample parameters used in the numerical simulations and are listed in the caption of Fig. 3.1.

Subplot (A) shows the behavior at operating point A, which lies inside the aS zone. Panel (A-1) shows sample trajectories in the A plane. The resultant 1D map, which is plotted in panel (A-2), has a single fixed point corresponding to a single locally-stable LC. The time evolution seen in panel (A-3) was obtained by numerically integrating the coupled stochastic equations of motion (2.4) and (2.7). The trace is then compared to experimental data taken from the same working point [panel (A-4)].

At operating point B [see Figs. 3.2 and 3.3 (B)] coexistence of an LC and an SC steady state occurs. The LC corresponds to the locally stable fixed point of the 1D map seen in panel (B-2). On the other hand, all initial points on the circle $|A|^2 = E_n$ that never return to it evolve towards the SC steady state $A_s$. Numerical time evolution shows noise-induced transitions between the two metastable solutions [panel (B-3)]. Experimental data for the same working point exhibits similar behavior [panel (B-4)]. At operating point C [see Fig. 3.2 and 3.3 subplot (C)] the LC has been annihilated by a discontinuity-induced bifurcation [54] and consequently only steady state response is observed.

The noise-induced transitions demonstrated in working point B can be understood as follows: in general, each solution (steady state or LC) can be characterized by a basin of attraction in the 3-dimensional phase space of the system (which has 3 coordinates $Re(A)$, $Im(A)$, and $T$). A transition between different solutions occurs when the system exits the basin of attraction of an initial solution due to fluctuations induced by external noise.

To further study noise-induced transitions, we fixed $\omega_p$ and vary $P_p$ starting from the MS(SC) zone $P_p = -26.7 \text{ dBm}$ to the aS zone $P_p = -25.6 \text{ dBm}$ [see the vertical line in Fig. (3.1)], and took relatively long time traces of the reflected power [similar to those seen in Fig. 3.3 (A-4), (B-4) and (C-4)]. The average lifetime of both the LC and the SC steady state, namely the average time the system is in one solution before making a transition to the other, were determined from these traces. This
3.5. ADDITIONAL CASES

Figure 3.5: Experimental data of life time of LC (circles) and steady state (crosses) compared to numerical predictions (solid and dashed lines respectively). The measurements above −28.5 dBm saturate to 1 μsec as this is the maximal measurement time.

data, compared to numerical simulation prediction (using the parameters listed in the caption of Fig. 3.1) is shown in Fig. 3.5. While the problem of the lifetime calculation of a steady state solution has been thoroughly studied for the case of smooth systems [58], very little is currently known about the lifetime of LC solutions, or lifetime in non-smooth systems [59].

3.5 Additional Cases

In spite of its simplicity, our model, as was demonstrated above, can successfully reproduce many of the experimental observations. However, as we point out below, some of the results were left unaccounted for. In another experiment using a similar device we observe the intermittency of two different LCs (see Fig. 3.6). Panel (A) shows the spectrum analyzer measurement of the reflected power as a function of the pump power $P_p$. Two distinct LCs having frequencies $f_1 \simeq 60$ MHz and $f_2 \simeq 80$ MHz are observed. For low pump powers ($P_p < -33.5$ dBm) only one LC at frequency $f_1$ is visible. In the range of $-33.55$ dBm $< P_p < -33.35$ dBm both LCs are seen, whereas for high pump power $P_p > -33.5$ dBm only one LC at frequency $f_2$ is seen. Panel (B), which shows a cross section of panel (A) at pump power of $P_p = -33.5$ dBm [indicated by a dashed line in panel (A)], demonstrates the behavior in the intermediate region, where both LCs are observed. Panel (C) shows the transitions in the time domain corresponding to the same pump power $P_p = -33.5$ dBm.

In general, intermittency of two (or more) different LCs can be theoretically reproduced using our simple model. However, we were unable to numerically obtain this
Figure 3.6: (Color) Experimental demonstration of intermittency between two LCs. Panel (A) shows a spectrum analyzer measurement of the reflected power $P_{\text{ref}}$ as a function of the offset frequency $\Delta f$ (with respect to the pump frequency $\omega_p/2\pi = 6.61 \text{GHz}$) and the pump power $P_p$. Panel (B) shows a cross section of panel (A) obtained at the value of $P_p = -33.5 \text{ dBm}$, which is indicated by a dashed line. The frequencies $f_1$ and $f_2$ of the two LCs are indicated by arrows. Panel (C) shows a time trace of the reflected power taken at the same value of $P_p$.

behavior without significantly varying some of the system’s experimental parameters. This discrepancy between experimental and theoretical results suggest that a further theoretical study is needed in order to develop a more realistic description of the system. Such description would have to exclude some of the simplifying assumptions that were made to derive our model.
Chapter 4

ESR Detection

4.1 Introduction

Electron spin resonance (ESR) is a well-known method enabling direct measurements of the electron spin Hamiltonian, with applications ranging from biology to materials science and physics [60, 61, 62]. However, a significant drawback of conventional ESR is its relatively low sensitivity compared with other spectroscopic and analytic techniques, such as fluorescence and mass spectrometry. For example, the world record in electron spin sensitivity stands today at \( \sim 10^6 \) spins per 1 sec of acquisition (often denoted as spins/\( \sqrt{\text{Hz}} \)) or slightly more than \( 10^4 \) spins in a reasonable \( \sim 1 \) h of acquisition [63], which is still far from single-electron spin sensitivity. This current sensitivity limitation also restricts the available imaging resolution of heterogeneous samples. Thus, while the laws of physics do not set a limit to the spatial resolution of ESR (at least up to the atomic-length scale), in practice, as the image’s voxel (volumetric pixel) size decreases, it contains less and less spins and thus quickly runs into the sensitivity limitation wall. For example, the systems in our laboratory, achieved recently a 440 nm resolution, limited mostly by spin sensitivity [64].

The above-mentioned numbers for sensitivity and resolution refer to ESR systems that employ "induction detection", that is, they make use of Faraday’s law for the detection of ESR signals by means of a pick-up coil or a microwave (MW) resonator. Induction detection is the basic principle behind all commercial state-of-the-art ESR systems; it enables the acquisition of high-resolution spectroscopic data with complex pulse sequences; facilitates the use of efficient imagining methodologies (meaning that signals are acquired and averaged in a parallel fashion from the entire sample); and features convenient sample handling. While our work is focused on induction detection ESR, other groups have looked into alternative detection methods in an attempt to increase sensitivity and resolution. These include, for example, magnetic resonance force microscopy [65], scanning tunneling microscopy ESR [66], spin-polarized scanning tunneling microscopy [67], electrically-detected magnetic resonance [68], and
indirect spin detection via diamond nitrogen-vacancy centers [69]. While these techniques may improve even more in the future, they have some inherent limitations, that result in limited applicability.

It is therefore evident that there is still a strong need to greatly improve the sensitivity of induction detection ESR up to the ultimate single-spin sensitivity (in a reasonable acquisition time of ~ 1 h or less), making it a generally applicable method of noninvasive detection and imaging of small numbers of electron spins. Here we take a step in that direction and show that the sensitivity of induction detection may be enhanced if it is employed in conjunction with our non linear superconducting resonators. In our scheme, the sample’s resonance properties affect the non linear properties of the resonator, thereby resulting in a different approach to the detection of ESR signals. Our experimental results demonstrate this approach in practice and are accompanied by a theoretical analysis explaining our observations. The measured responsivity of our resonator is enhanced by a factor of up to 100 when operating the system in the nonlinear regime.

4.2 ESR Theory

In ferromagnetic materials under external magnetic field, induced magnetic dipole moments are created; these dipoles are a result of the change in spin states of free electrons in the material (referred hereafter as the free spins). The total induced magnetic field in the material (the macroscopic value of the effect) is called magnetization. For a material under a constant magnetic field, the magnetization can be written as [62]:

\[ M_0 = \frac{1}{\mu_0} \chi_0 B_0, \]  

where \( M_0 \) is the constant magnetization, \( B_0 \) is the external magnetic field, \( \mu_0 \) is the permeability of vacuum, \( \chi_0 \) is the static magnetic susceptibility. The value of \( \chi_0 \) which is dependent on the material, is a measurement for the strength of the magnetization, and can be calculated by:

\[ \chi_0 = \frac{\mu_0 N \gamma_{ESR}^2 h^2}{4k_B T_0}, \]  

where \( N \) is the density of the free spins, \( \gamma_{ESR} = g\mu_B \) is the g-factor, with \( g \approx 2 \), \( \mu_B \) is the Bohr magneton, \( k_B \) is the Boltzmann constant, and \( T_0 \) is the temperature of the material.

Under a time varying magnetic field and without dissipation, the magnetization equation of motion is given by:

\[ \frac{dM}{dt} = \gamma_{ESR} M \times B. \]
4.2. ESR Theory

Eq. 4.3 is an oscillatory equation and does not account for any dissipation effects in the material. In 1946 Félix Bloch introduced a phenomenological equations to account for the dissipation in Eq. 4.3. The modified equations introduced are [70]:

\[
\frac{dM_x}{dt} = \gamma_{ESR} (M_y B_z - M_z B_y) - \frac{M_x}{t_2},
\]

(4.4a)

\[
\frac{dM_y}{dt} = \gamma_{ESR} (M_z B_x - M_x B_z) - \frac{M_y}{t_2},
\]

(4.4b)

\[
\frac{dM_z}{dt} = \gamma_{ESR} (M_x B_y - M_y B_x) - \frac{M_z - M_0}{t_1},
\]

(4.4c)

where \(t_1\) and \(t_2\) are the ESR longitudinal and horizontal relaxation times respectively. The longitudinal relaxation time is associated with the spin-lattice interaction, and describes the dissipation of angular momentum and thus magnetization. For an ensemble of free spins under a DC magnetic field \(B_0 \hat{z}\) and which are prepared in the upper energy level \(|+z\rangle\), the time \(t_1\) describes the exponential decay to the lower spin level \(|-z\rangle\). The horizontal relaxation times is associated with the spin-spin relaxation, and describes the spins dephasing. For an ensemble of spins which are all prepared with the same initial wave function phase, \(t_2\) describes the exponential decay toward a state where each spin has a random wave function phase (uncorrelated with other spins).

The CW ESR experiment is done in the following manner: The ESR sample is placed close to a microwave resonator, and the two are placed inside an electro magnet. The electro magnet creates a DC magnetic field in the Z direction \(B_z = B_0 \hat{z}\). The magnetic field of the resonator is rotating in the X-Y plane, and is expressed by \(B_x = B_1 \cos(\omega_0 t) \hat{x}\) and \(B_y = -B_1 \sin(\omega_0 t) \hat{y}\), where \(B_1 \ll B_0\). Eq. 4.4 can now be written in the following manner:

\[
\frac{dM_x}{dt} = \gamma_{ESR} (M_y B_0 + M_z B_1 \sin(\omega_0 t)) - \frac{M_x}{t_2},
\]

(4.5a)

\[
\frac{dM_y}{dt} = \gamma_{ESR} (M_z B_1 \cos(\omega_0 t) - M_x B_0) - \frac{M_y}{t_2},
\]

(4.5b)

\[
\frac{dM_z}{dt} = -\gamma_{ESR} (M_x B_1 \sin(\omega_0 t) + M_y B_1 \cos(\omega_0 t)) - \frac{M_z - M_0}{t_1}.
\]

(4.5c)

We define two frequencies characterizing the system: the Larmor frequency \(\omega_l = \gamma_{ESR} B_0\) and the Rabi frequency \(\omega_r = \gamma_{ESR} B_1\). For a typical ESR experiment, we can assume that \(2\pi/\omega_l, 2\pi/\omega_r \ll t_1, t_2\) (values for \(\omega_l, \omega_r, t_1\) and \(t_2\) in our experiment are given in the caption of Fig. 4.4). We now change the coordinates of Eqs. 4.5a and 4.5b to a rotating frame of reference:

\[
M'_x = M_x \cos(\omega_0 t) - M_y \sin(\omega_0 t),
\]

(4.6a)

\[
M'_y = M_x \sin(\omega_0 t) + M_y \cos(\omega_0 t),
\]

(4.6b)
The equations of motion in the rotating frame will take the form:

\[ \frac{dM'_x}{dt} = -(\omega_p - \omega_l)M'_y - \frac{M'_x}{t_2}, \quad (4.7a) \]
\[ \frac{dM'_y}{dt} = +(\omega_p - \omega_l)M'_x - \frac{M'_y}{t_2} + \omega_i M_z, \quad (4.7b) \]
\[ \frac{dM_z}{dt} = -\omega_i M'_y - \frac{M_z - M_0}{t_1}. \quad (4.7c) \]

The steady state solution for Eq. 4.7 \((t \gg t_1, t_2)\) is:

\[ M'_x = \frac{\omega_l t_2^2(\omega_p - \omega_l)}{1 + t_2^2(\omega_p - \omega_l)^2 + \omega_i^2 t_1 t_2} M_0, \quad (4.8a) \]
\[ M'_y = \frac{\omega_l t_2}{1 + t_2^2(\omega_p - \omega_l)^2 + \omega_i^2 t_1 t_2} M_0, \quad (4.8b) \]
\[ M_z = \frac{1 + \omega_l t_2^2(\omega_p - \omega_l)}{1 + t_2^2(\omega_p - \omega_l)^2 + \omega_i^2 t_1 t_2} M_0. \quad (4.8c) \]

The effective on the ESR sample susceptibility can then be written as \([62, 71]\):

\[ \chi = \chi' + \chi'' \]

with

\[ \chi' = -\frac{1}{2} \frac{(\omega_p - \omega_l)t_2^2}{1 + (\omega_p - \omega_l)^2 t_2^2 + \omega_i^2 t_1 t_2} \omega_l \chi_0, \quad (4.9a) \]
\[ \chi'' = -\frac{1}{2} \frac{t_2}{1 + (\omega_p - \omega_l)^2 t_2^2 + \omega_i^2 t_1 t_2} \omega_l \chi_0. \quad (4.9b) \]

The change in the susceptibility of the ESR sample gives rise to changes in the resonator’s resonance frequency, \(\omega_0\), the damping coefficient, \(\gamma\), and the quality factor, \(Q\), according to the expressions \([60]\):

\[ \frac{\Delta \omega}{\omega_0} \approx \zeta \chi', \quad (4.10a) \]
\[ \frac{\Delta Q}{Q_0} \approx -\frac{\Delta \gamma}{\gamma} \approx -Q_0 \zeta \chi'', \quad (4.10b) \]

where \(\zeta\) is the filling factor of the sample \([60]\).

### 4.3 Experimental Setup

For the ESR measurements we used a stripeline YBCO-on-sapphire RF resonator whose design and fabrication are described in chapter 2. The resonator was covered by a sapphire wafer. It was packed in a copper box whose inner top and bottom surfaces were covered with niobium to provide superconducting ground planes for the
4.3. EXPERIMENTAL SETUP

Figure 4.1: Experimental setup for ESR signal measurements. The device is installed in a cryostat where a strong static magnetic field $B_z = B_0 \hat{z}$ is applied. A single coherent tone, with angular frequency $\omega_0$ and power $P_0$, is injected into the feedline. The reflected signal is split and measured by a spectrum analyzer in the frequency domain, and by a lock-in amplifier (LIA). The LIA is tuned to the frequency of the slow magnetic field modulation $b_z = b_z \cos(\omega_{LIA} t) \hat{z}$ that is applied through a wire. The inset shows an electron micrograph of the microbridge.

The experimental setup presented in Fig. 4.1 is similar to the one presented in chapter 2.3 (Fig. 2.3). The device was fully immersed in liquid helium inside the core of a superconducting coil. The ESR signal was measured in continuous wave (CW) mode, similar to any conventional ESR system, with the exception that automatic frequency control was not used to track the resonator’s resonance frequency. A monochromatic MW signal at a frequency close to the resonance frequency was injected. The reflected signal was measured while the external magnetic field was slowly scanned. In another setup, the resonator $|S_{11}|$ reflection coefficient was measured using a network analyzer. The resonator’s loaded Q factor was found to be in the order of 5000.
Figure 4.2: (Color) Panel a: Self-modulation stability diagram. The stability map, which is obtained by Eq. (4.13) contains 4 stability zones: the superconducting mono stable zone (MS(SC)) the normal conducting mono stable zone (MS(NC)) the bi stable zone (BiS) and the astable zone (aS). We compare the case were \( B_0 \) is far from \( B_0^{\text{ESR}} \) (solid-blues lines) and the case of \( B_0 = B_0^{\text{ESR}} \) (dashed-red lines). The experiment in Fig. 4.4 is performed along the black line, at an input power range \(-3 \text{ dBm} < P_p < -1 \text{ dBm} \), and slightly above the resonance frequency. Panels b-c: Data from spectrum analyzer in the monostable (c, \( P_p = -3 \text{ dBm} \)) and astable (b, \( P_p = -1 \text{ dBm} \)) zones, when \( B_0 \) is far from \( B_0^{\text{ESR}} \).

4.4 ESR Measurements

To examine the ERS response, we examined the resonator’s response in two regimes: the linear response, namely when the resonator is excited below the SM threshold, and the non-linear response, namely when the resonator is excited just on the SM threshold. As will be shown below, ESR responsivity can be greatly enhanced when operating the resonator close to the SM threshold.

We began by examining the resonator’s linear response to the ESR signal. The resonator was excited with power well below the SM threshold, while the static magnetic field, \( B_z = B_0 \hat{z} \), was slowly scanned (0.01 T/min). We measured the reflection coefficient of the resonator as a function of the input frequency. Near a magnetic field of \( B_0^{\text{ESR}} = 0.215 \text{ T} \) the resonance frequency of the resonator was shifted and the Q-factor decreased (see Fig. 4.3). The results obtained by us are similar to those presented by others dealing in the coupling of a paramagnetic sample and a linear superconducting resonator [72, 73, 74].

We then examined the response of the resonator at the non-linear regime. We excited the resonator by injecting a monochromatic tone at frequency \( \omega_p \) which is close to the resonance frequency, and power in the vicinity of the modulation threshold. The static magnetic field was slowly ramped around \( B_0^{\text{ESR}} \), while a low-frequency
(1.23 kHz) low amplitude (20 μA) AC current was injected into the wire (as in conventional CW induction detection schemes) to provide the magnetic field modulation. A lock-in amplifier (LIA) was tuned to the slow modulation frequency and measured the envelope of the signal reflected off the resonator. Slightly below the modulation threshold, the response was linear and the power reflected off the resonator was proportional to the $|S_{11}|$ parameter (the reflection coefficient); therefore the LIA measured the derivative of the $|S_{11}|$ parameter with respect to the magnetic field. The ESR-induced change in the LIA signal was seen at the same value of $B_{0}^{\text{ESR}}$ as in the direct $|S_{11}|$ measurement (Fig. 4.4-b, blue line). When the input power was set exactly at the modulation threshold, the LIA signal increased significantly by a factor of up to 100 (Fig. 4.4-b, green line). As we scanned the static magnetic field we found that the non-linearity power threshold shifted; at $B_{0}^{\text{ESR}}$ the power threshold increased by $\sim 1.2$ dB relative to the case where the static field is far from the resonance value. Thus, by scanning over the $P_{p} - B_{0}$ plane, and measuring the LIA signal we can obtain the ESR spectrum. (see Fig. 4.4-a).

4.5 Results Analysis

To analyze the results, we consider an ESR sample placed near a current anti node in a stripline MW resonator embedded with a microbridge. An external static magnetic field tunes the Larmor frequency of the ESR sample close to the resonator’s resonance frequency. As discussed in section 2.7 the dynamics of the resonator can be captured by two coupled equations of motion (see Eqs. (2.4) and (2.7)):

$$\frac{dA}{dt} = [i(\omega_{p} - \omega_{0}) - \gamma] A - i\sqrt{2\gamma_{1}a^{in} + c^{in}}, \quad (4.11)$$

and

$$C\frac{dT}{dt} = -H(T - T_{0}) + 2h\omega_{p}\gamma_{2}\alpha |A|^2, \quad (4.12)$$

where we will now denote $\gamma = \gamma_{1} + \gamma_{2}$, $\gamma_{1}$ is the coupling constant between the resonator and the feedline, and $\gamma_{2} + \gamma_{3}$ is the damping rate of the mode, where $\gamma_{2}$ denotes the damping rate of the microbridge.

As seen in chapter 3, the two coupled Eqs. (4.11) and (4.12), have two attractor sets [76]: $\{A_{n}, T_{n}\}$ (the "super attractor") and $\{A_{n}, T_{n}\}$ (the "normal attractor"). Consequently, four stability zones can be identified in the plane of pump power $P_{p} = h\omega_{p}|a^{in}|^2$ vs. pump frequency $\omega_{p}$ (see Fig. 4.2).

The borderlines between the four stability zones are given by:
Figure 4.3: (Color) (A) $|S_{11}|$ measurements under the external magnetic field scan. Measurements are plotted for increasing magnetic field with intervals of 0.001 T, and are shifted by 0.5 dB for clarity. At magnetic field of $B_0^{\text{ESR}} = 0.215$ T (in red) a shift in the resonance frequency along with a Q factor decrease is seen. (B) and (C) The resonator’s resonance frequency and Q factor extracted from the $|S_{11}|$ measurements in inset (A). The experimental results (solid blue line) are compared with theoretical prediction based on Eq. (4.10) (dashed red line). The parameters used in Eq. (4.9) are: $t_1 = t_2 = 1/61$ MHz/π, $N = 2 \cdot 10^{27}$ spins/m$^3$ [71], $T_0 = 4.2$ K, and $\zeta = 8.6 \cdot 10^{-5}$. The simulation takes into consideration a single homogenous broadened ESR line, while in the experimental data g-factor anisotropy and hyperfine effects are observed [75].
4.5. RESULTS ANALYSIS

Figure 4.4: (Color) ESR spectrum obtained in the linear and non-linear regimes. (a) The signal reflected off the resonator, as a function of the static magnetic field and the input power, measured using a LIA at 1.23 kHz. The ESR spectrum can be obtained from the non-linear response readings (local maxima of the graph), and is compared to analytical results obtained from Eqs. (4.13a) and (4.10) (dashed white line). The fitting parameters are $H = 7\mu$ W/K and $T_c - T_0 = 70$ K. (b) Cuts from the two dotted lines which are marked in the colormap: −2.8 dBm (solid-blue) and −2.2 dBm (solid-green) input power, showing linear and non linear response respectively. Note the discontinuity in the vertical axis, which was created in order to make visible the relatively small change in the linear regime.
\[ P_p = \frac{H(T_c - T_0)}{4\gamma_1 \gamma_2 s \alpha} \left[ (\omega_p - \omega_{0,s})^2 + \gamma_n^2 \right], \quad (4.13a) \]

\[ P_p = \frac{H(T_c - T_0)}{4\gamma_1 \gamma_2 n \alpha} \left[ (\omega_p - \omega_{0,n})^2 + \gamma_n^2 \right]. \quad (4.13b) \]

Eq. (4.13a) determines the border line between the super conducting mono stable zone and the astable zone which is the modulation threshold. The experiments were conducted in the vicinity of this borderline (Fig. 4.2-a).

As discussed in Sec. 4.2 the ESR induces a change in the resonance frequency and Q-Factor (Eqs. 4.10). In the linear regime, a comparison between the analytical results and the experimental results in Fig. 4.3, shows good agreement. As can be seen from Eq. (4.13a) the ESR shifts the borderline between the mono-stable and astable zones. Again, when comparing the analytical results with the experimental results in Fig. 4.4, good agreement is obtained.

### 4.6 ESR Non-Linear vs. Linear Measurements

A potential advantage of our methodology can be explained by comparing it to the conventional linear method. In induction detection ESR with a resonator having a linear response the output signal is relatively small and consequently various readout elements such as cryogenic amplifiers, LIAs, etc. are commonly employed. Reducing the noise of such readout elements to cryogenic temperatures is a major engineering challenge [77, 78]. In our scheme, on the other hand, ESR signal amplification is achieved by the resonator’s intrinsic behavior, and consequently no further active amplification is needed. A second potential advantage of our methodology is important in the case of paramagnetic materials with broad resonance lines. This advantage can be understood in relation to LIA measurement and field modulation. Commonly, LIA measurement is applied in order to reduce 1/f noise. In the present experiment the LIA detection is carried out with respect to a modulation in the static magnetic field with low frequency. This modulation brings the resonator in/out of the nonlinearity boundary. Such detection scheme is the most common in CW ESR and facilitates its high sensitivity. However, for paramagnetic materials having broad resonance lines reaching 0.1 T [60, 61] and above, conventional field modulation at a large amplitude is not possible, because it causes excessive heat and mechanical vibrations. This problem can be solved in our scheme by employing amplitude modulation of the driving MW frequency (instead of field modulation), for which the width of the ESR spectrum does not limit the detection (see Fig. 4.2).
Chapter 5

Self-Modulation for Two-Mode Coupling

5.1 Introduction

In some of our resonators, we found two resonance frequencies which are close to each other, namely, for the two resonance frequencies $\omega_1$, $\omega_2$ we find that $\omega_n \gg |\omega_1 - \omega_2|$ for $n = 1, 2$. In these cases, the basic SM model introduced earlier could not fully explain the behavior observed in experiments. In this chapter we develop an extended model, which accounts for two modes or more in the the same resonator. We then show our experimental data and compare them to the theory.

5.2 Hamiltonian and Equations of Motion of the Modes

Assume a general RF resonator that has two modes. Each mode is coupled to two ports: (1) a common port which represents a feedline through which signals are injected into the resonator, and (2) a separate port, which represents the damping in that mode. The two ports are modeled as semi-infinite transmission lines [79]. The total Hamiltonian of the system reads:
$H = \hbar \omega_1 A_1^\dagger A_1 + \hbar \omega_2 A_2^\dagger A_2$
\[+ \int d\omega \hbar \omega a_0^\dagger (\omega) a_0 (\omega)\]
\[+ \int d\omega \hbar \omega a_1^\dagger (\omega) a_1 (\omega)\]
\[+ \int d\omega \hbar \omega a_2^\dagger (\omega) a_2 (\omega)\]
\[+ \hbar \sqrt{\frac{\gamma_{01}}{\pi}} \int d\omega \left( e^{i\phi_{01}} A_1^\dagger a_0 (\omega) + e^{-i\phi_{01}} a_0^\dagger (\omega) A_1 \right)\]
\[+ \hbar \sqrt{\frac{\gamma_{02}}{\pi}} \int d\omega \left( e^{i\phi_{02}} A_2^\dagger a_0 (\omega) + e^{-i\phi_{02}} a_0^\dagger (\omega) A_2 \right)\]
\[+ \hbar \sqrt{\frac{\gamma_{11}}{\pi}} \int d\omega \left( e^{i\phi_{11}} A_1^\dagger a_1 (\omega) + e^{-i\phi_{11}} a_1^\dagger (\omega) A_1 \right)\]
\[+ \hbar \sqrt{\frac{\gamma_{22}}{\pi}} \int d\omega \left( e^{i\phi_{22}} A_2^\dagger a_2 (\omega) + e^{-i\phi_{22}} a_2^\dagger (\omega) A_2 \right),\]

(5.1)

where $A_1$ ($A_1^\dagger$) and $A_2$ ($A_2^\dagger$) are the creation (annihilation) operators in the resonator, $a_0 (\omega)$ ($a_0^\dagger (\omega)$) are the creation (annihilation) operators for the feedline port, and $a_n (\omega)$ ($a_n^\dagger (\omega)$) are the creation (annihilation) operators for the damping ports, where $n = 1, 2$.

We now generate the Heisenberg equations of motion according to

$$i\hbar \frac{dO}{dt} = [O, H],$$

(5.2)

where $O$ is an operator and $H$ is the total Hamiltonian. Hence, using the commutation relation

$$[A_1, A_1^\dagger] = [A_2, A_2^\dagger] = 1,$$

(5.3)

the result is

$$\frac{dA_1}{dt} = -i\omega_1 A_1 - i\sqrt{\frac{\gamma_{01}}{\pi}} e^{i\phi_{01}} \int d\omega a_0^\dagger (\omega) - i\sqrt{\frac{\gamma_{11}}{\pi}} e^{i\phi_{11}} \int d\omega a_1 (\omega),$$

(5.4)

$$\frac{dA_2}{dt} = -i\omega_2 A_2 - i\sqrt{\frac{\gamma_{02}}{\pi}} e^{i\phi_{02}} \int d\omega a_0^\dagger (\omega) - i\sqrt{\frac{\gamma_{22}}{\pi}} e^{i\phi_{22}} \int d\omega a_2 (\omega).$$

(5.5)

The use of the commutation relations (with $n, n' = 0, 1, 2$):

$$[a_n (\omega), a_{n'}^\dagger (\omega')] = \delta (\omega - \omega') \delta_{n,n'},$$

$$[a_n (\omega), a_{n'} (\omega')] = 0,$$

(5.6)
5.2. Hamiltonian and Equations of Motion of the Modes

produces
\[
\frac{da_0(\omega)}{dt} = -i \omega a_0(\omega) - i \sqrt{\frac{\gamma_{01}}{\pi}} e^{-i\phi_{01}} A_1 - i \sqrt{\frac{\gamma_{02}}{\pi}} e^{-i\phi_{02}} A_2. \tag{5.7}
\]

and for other bath modes, \(a_n\) where \(n = 1, 2\):
\[
\frac{da_n(\omega)}{dt} = -i \omega a_n(\omega) - i \sqrt{\frac{\gamma_{nn}}{\pi}} e^{-i\phi_{nn}} A_n. \tag{5.8}
\]

Integrating the equations of motion for the baths yields
\[
a_0(\omega) = e^{-i\omega(t-t_0)} a_0(t_0, \omega) - i \sqrt{\frac{\gamma_{01}}{\pi}} \int_t^{t_0} e^{-i\omega(t-t')} A_1(t') dt' - i \sqrt{\frac{\gamma_{02}}{\pi}} \int_t^{t_0} e^{-i\omega(t-t')} A_2(t') dt', \tag{5.9}
\]

and for \(n = 1, 2\)
\[
a_n(\omega) = e^{-i\omega(t-t_0)} a_n(t_0, \omega) - i \sqrt{\frac{\gamma_{nn}}{\pi}} e^{-i\phi_{nn}} \int_t^{t_0} e^{-i\omega(t-t')} A_n(t') dt'. \tag{5.10}
\]

We now substitute these into the equations of motion for \(A_1\) (and the similar equation for \(A_2\)):
\[
\frac{dA_1}{dt} = -i \omega_1 A_1 - i \sqrt{\frac{\gamma_{01}}{\pi}} \int d\omega e^{-i\omega(t-t_0)} a_0(t_0, \omega) - \gamma_{01} \int d\omega \int_t^{t_0} e^{-i\omega(t-t')} A_1(t') dt' - i \sqrt{\frac{\gamma_{11}}{\pi}} e^{i\phi_{01}} \int d\omega e^{-i\omega(t-t_0)} a_1(t_0, \omega) - \gamma_{11} \int d\omega \int_t^{t_0} e^{-i\omega(t-t')} A_1(t') dt'. \tag{5.11}
\]

Using the following relations
\[
\int d\omega e^{-i\omega(t-t')} = 2\pi \delta(t - t'), \tag{5.12}
\]
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\[ \int_{t_0}^{t} \delta(t - t') f(t') \, dt' = \frac{1}{2} \text{sgn}(t - t_0) f(t), \]  
(5.13)

where \( \text{sgn}(x) \) is the sign function

\[ \text{sgn}(x) = \begin{cases} 
+1 & \text{if } x > 0 \\
-1 & \text{if } x < 0 
\end{cases} \],  
(5.14)

and introducing the incoming modes (with \( n = 0, 1, 2 \))

\[ a_n^{\text{in}}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega(t-t_0)} a_n(t_0, \omega), \]  
(5.15)

the result is

\[ \frac{dA_1}{dt} = -i\omega_1 A_1 - (\gamma_{01} + \gamma_{11}) A_1(t) - i\sqrt{2\gamma_{01} \epsilon_{01}} a_0^{\text{in}}(t) - i\sqrt{2\gamma_{11} \epsilon_{11}} a_1^{\text{in}}(t) - \sqrt{\gamma_{01} \gamma_{02} e^{i(\phi_{01} - \phi_{02})}} A_2(t). \]  
(5.16)

Similarly for \( A_2 \)

\[ \frac{dA_2}{dt} = -i\omega_2 A_2 - (\gamma_{02} + \gamma_{22}) A_2(t) - i\sqrt{2\gamma_{02} \epsilon_{02}} a_0^{\text{in}}(t) - i\sqrt{2\gamma_{22} \epsilon_{22}} a_2^{\text{in}}(t) - \sqrt{\gamma_{01} \gamma_{02} e^{i(\phi_{01} - \phi_{02})}} A_1(t). \]  
(5.17)

5.3 Equations of Motion under the Slow Envelope Approximation

Assume that the resonator is driven by a coherent tone, namely

\[ a_0^n(t) = (a_0^n + i\eta_0^n) \exp(-i\omega_p t), \]  
(5.18)

where \( a_0^n \) is a complex amplitude such that \( |a_0^n|^2 \propto P_p \), and \( \eta_0^n \) is a noise term. Assume further that the only input from the two other ports (which represents the damping) is due to noise, and employ the notation \( a_0^n = \eta_0^n \exp(-i\omega_p t) \).

The modes in the resonator can be then written in the form \( A_n(t) = \eta_n(t) \exp(-i\omega_p t) \), where \( \eta(t) \) is a complex amplitude which is assumed to vary slowly on time scales of \( 1/\omega_p \). The equations of motion then read as follows:

\[ \frac{dA_1}{dt} = i(\omega_p - \omega_1) A_1 - (\gamma_{01} + \gamma_{11}) A_1(t) - i\sqrt{2\gamma_{01} \epsilon_{01}} \eta_0^n(t) - \sqrt{\gamma_{01} \gamma_{02} e^{i(\phi_{01} - \phi_{02})}} A_2(t) + i\eta_1^n, \]  
(5.19)
and 
\[
\frac{dA_2}{dt} = i(\omega_p - \omega_2)A_2 - (\gamma_{02} + \gamma_{22})A_2(t) \\
- i\sqrt{2\gamma_{02}}e^{i\phi_{02}}d_{i2}^n(t) \\
- \sqrt{2\gamma_{02}\gamma_{12}}e^{i(\phi_{02} - \phi_{01})}A_1(t) \\
+ c_{H}^2,
\]
(5.20)

where \(c_n^m = c_{0}^{m} + c_{2}^{m}\), is the total noise term, that represents an input Gaussian noise, whose time autocorrelation function is given by
\[
\langle c_n^m(t)c_n^{m*}(t') \rangle = G\omega_0\delta(t - t'),
\]
(5.21)

where the constant \(G\) can be expressed in terms of the effective noise temperature \(T_{\text{eff}}\) as
\[
G = \frac{2(\gamma_{0n} + \gamma_{nn})}{\omega_n} \frac{k_B T_{\text{eff}}}{\hbar \omega_n}.
\]
(5.22)

### 5.4 Thermal Balance Equation for the Bridge

The dissipated power, \(Q_n\), in mode \(n\) \((n = 1, 2)\) of the resonator is given by
\[
Q_n = \hbar \omega_n 2\gamma_{2n} |A_n|^2.
\]
(5.23)

We assume nonlinearity which originates in a local hot-spot in the resonator. If the hot-spot is assumed to be sufficiently small, its temperature \(T\) can be considered as homogeneous. The temperature of other parts of the resonator is assumed to be equal to that of the coolant, \(T_0\). The total power \(Q\) heating up the hot spot is given by
\[
Q = \alpha_1 Q_1 + \alpha_2 Q_2
\]
(5.24)

where \(0 \leq \alpha_n \leq 1\). The power transferred from the hot-spot to the coolant is given by the term \(W = H(T - T_0)\), where \(H\) is the heat transfer coefficient. The heat balance equation reads
\[
C\frac{dT}{dt} = Q - W,
\]
(5.25)

where \(C\) is the heat capacitance of the hotspot.

### 5.5 Coupling of the Equations of Motion

The coupling between Eqs. (5.19),(5.20) and Eq. (5.25) is due to the dependence of the resonance frequencies \(\omega_n\) and the damping rates \(\gamma_{nn}\) of the driven modes, on the impedance of the microbridge [57], which in turn depends on the temperature of the
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hotspot [37]. We assume the simplest case, where the dependence is a step-function dependence that occurs at the critical temperature \( T = T_c \), namely

\[
\omega_n, \gamma_{nn} = \begin{cases} 
\omega_{n,S}, \gamma_{nn,S} & T < T_c \\
\omega_{n,N}, \gamma_{nn,N} & T > T_c 
\end{cases},
\]

where the subscripts S and N represent superconductive and normal conductive phases, respectively.

5.6 Stationary Solution and Stability Zones

When looking for a stationary solution, we first assume a noiseless case, \( c_n^i = 0 \). Since Eqs. (5.19), (5.20) and (5.25) are piecewise linear, it is possible to find stationary solutions in the cases where the bridge is either superconductive or normal conductive. A stationary solution will be in the form of

\[
\lim_{t \to \infty} \{ A_1(t), A_2(t), T(t) \} = \{ A_{1,\sigma}, A_{2,\sigma}, T_\sigma \},
\]

where \( \sigma \) is one of two possible phases of the bridge (S or N). The stationary solution to Eqs. (5.19) and (5.20) is given by:

\[
M_\sigma \begin{pmatrix} A_{1,\sigma} \\ A_{2,\sigma} \end{pmatrix} = \begin{pmatrix} i \sqrt{2\gamma_{11,\sigma} e^{i\phi_{01}}} \\ i \sqrt{2\gamma_{22,\sigma} e^{i\phi_{02}}} \end{pmatrix} a_0^i,
\]

where

\[
M_\sigma = \begin{pmatrix} i(\omega_P - \omega_{1,\sigma}) - (\gamma_{11} + \gamma_{11,\sigma}) & -\sqrt{\gamma_{11}\gamma_{22}} e^{i(\phi_{01} - \phi_{02})} \\ -\sqrt{\gamma_{11}\gamma_{22}} e^{i(\phi_{01} - \phi_{02})} & i(\omega_P - \omega_{2,\sigma}) - (\gamma_{22} + \gamma_{22,\sigma}) \end{pmatrix}.
\]

Hence

\[
\begin{pmatrix} A_{1,\sigma} \\ A_{2,\sigma} \end{pmatrix} = M_\sigma^{-1} \begin{pmatrix} i \sqrt{2\gamma_{11} e^{i\phi_{01}}} \\ i \sqrt{2\gamma_{22} e^{i\phi_{02}}} \end{pmatrix} a_0^i.
\]

The stationary solution of Eq. (5.25) is given by:

\[
\frac{H}{\hbar} (T_\infty - T_0) = \frac{\sqrt{\alpha_1^4 \gamma_{11,\sigma} \omega_{1,\sigma}} A_{1,\sigma}}{\sqrt{\alpha_2^4 \gamma_{22,\sigma} \omega_{2,\sigma}}} \begin{pmatrix} i \sqrt{\alpha_1^2 \gamma_{11,\sigma} \omega_{1,\sigma}} A_{1,\sigma} \\ i \sqrt{\alpha_2^2 \gamma_{22,\sigma} \omega_{2,\sigma}} A_{2,\sigma} \end{pmatrix} M_\sigma^{-1} M_\sigma^{-1} \begin{pmatrix} \sqrt{\alpha_1^4 \gamma_{11,\sigma} \omega_{1,\sigma}} \\ \sqrt{\alpha_2^4 \gamma_{22,\sigma} \omega_{2,\sigma}} \end{pmatrix} |a_0^i|^2.
\]
5.7. EXPERIMENTAL DATA OF MULTIMODE SM IN YBCO RESONATORS

The stationary solutions will be self-consistent only in the case where \( T_N > T_c \) if we assume a normal phase of the bridge or \( T_s < T_c \) if we assume a superconductive phase of the bridge. Therefore, the separatrix is on the line holding the condition \( T_\infty = T_c \):

\[
|a_0^{in}|^2 = \frac{H}{4\hbar} \left( \frac{T_c - T_0}{M^{-1}_\sigma M^{-1}_\sigma} \right) \left( \frac{T_0}{\sqrt{\frac{\alpha_1 \gamma_{11} \omega_{1,\sigma}}{\alpha_2 \gamma_{22} \omega_{2,\sigma}}}} \right) + \frac{1}{(\sqrt{\frac{\alpha_1 \gamma_{11} \omega_{1,\sigma}}{\alpha_2 \gamma_{22} \omega_{2,\sigma}}})} . \tag{5.32}
\]

5.7 Experimental Data of Multimode SM in YBCO Resonators

Figure 5.1 shows an example of two close resonance frequencies in interaction. Panel C shows \( S_{11} \) measurements of G3.2. At low input powers (\( \sim -10 \) dBm) two close resonance frequencies appear, at 5.68 GHz and 5.695 GHz, while for higher input powers, they get significantly broadened. Panel A shows the experimental SM diagram. For high input powers (\( \sim -10 \) dBm) and close to the two resonance frequencies, SM is seen. As can be seen by comparing Fig. 5.1 with Fig. 2.7, the behavior is significantly different; The single-mode model of Ref. [9] can not account for the data shown in Fig. 5.1, as it predicts parabola shaped separatrices. Figure 5.1-B shows the predicted normal and super separatrices for G3.2. Above the superconductive separatrix (solid-black line), super attractor is unstable, whereas below the normal separatrix (dashed-red line) the normal attractor is unstable. Between those two lines, the system cannot reach a stationary solution and is bounded to cycle between the two phases.
Figure 5.1: (Color) SM observed in G3.2 resonator, where two closely lying mode are seen. (A) SM diagram (B) Simulation of the the SM diagram outlines using Eq. (5.32). The solid-black line represents the superconductive separatrix and the dashed-red line represents the normal separatrix. (C) $S_{11}$ measurements. Lines indicate increasing input power from $-30$ dBm to 0 dBm in steps of 2.5 dBm. Lines are shifted by 0.1 dBm for clarity.
Chapter 6

On-Fiber Single-Photon Detectors

6.1 Introduction

Superconducting nanowire single-photon detectors (SNSPD) [20] are considered a promising technology for single photon detection in the visible-to-near-infra-red band. The detection mechanism, first introduced by Kadin et al. [19], utilizes a fast avalanche process, in which a single photon is absorbed in the SNSPD and creates a resistive section in the nanowire. In the following introduction we give a brief overview of this detection mechanism. We then proceed to define and describe two figures of merit of the device which were the focus for improvement of this work: the coupling efficiency and the absorption efficiency. We describe methods given in literature for improving these two aspects and their limitations. In the following sections this overview will be repeated with mathematical and physical formulation, together with the analysis of our new device fabrication approach. First we provide a theoretical analysis of the on-fiber detector and cavity-operation mode. We then describe in details the detector’s fabrication process and show the experimental results of on-fiber single-photon detection without the cavity.

An illustration of the SNSPD detection mechanism is given in Fig. 6.1 (taken from Ref. [20]). A superconducting nanowire is cooled below its critical temperature, and is biased with current close to its critical current. The nanowire is connected in parallel to a resistor, which is referred as the load resistor hereafter; the load resistor current is much smaller than the nanowire current, since the resistance of the wire and its contact pads is much smaller than load resistor resistance. The nanowire is then illuminated by some light source. Upon an absorption of a photon in the nanowire, the energy of the photon breaks many cooper-pairs, and thus creates a resistive area in the wire; this area is referred hereafter as the "hot-spot". As the electrical current in the wire is now routed around the hot spot, the current density in the side of the nanowire surpasses the critical current density; a resistive section is then created across the wire. At first the current flows in the wire’s normal conducting section,
Figure 6.1: SNSPD detection mechanism, from Ref. [20]. (a) A photon is absorbed in the nanowire. (b) A hotspot is created and the current is pushed to the sides of the hotspot. (c) The current density surpass the critical current density and a resistive section is formed. (d) The resistive section area grows due to Joule heating.

which results with Joule self-heating and fast increase in the resistive section area. Next, the current routes to the load resistor. As time passes, and there is no more Joule heating of the wire, the wire cools down and is ready for another detection event. The time constant that sets the device detection rate is $\tau = L_k / R_L$ where $L_k$ is the nanowire kinetic inductance, and $R_L$ is the load resistor resistance. The kinetic inductance is linearly proportional to the wire length [28], and thus the smaller the device is, the faster it is. A typical device made of 100 nm wide 5 nm thick niobium-nitride wires folded to an area of 25 $\mu m^2$, has a detection rate of 400 MHz [28, 29].

The detection efficiency $\eta$, is defined as the percentage of photons detected by the detector, out of those impinging on it. The detection efficiency can be calculated as $\eta = \eta_C \times \eta_A \times \eta_P$, where: the coupling efficiency $\eta_C$ is the percentage of photons that reach the detector out of those sent, the absorption efficiency $\eta_A$ is the percentage of photons absorbed in the detector out of those that reach it, and the pulse efficiency $\eta_P$ is the percentage of photons creating a pulse out of those absorbed.

Achieving high coupling efficiency requires focusing the input light on the detector, typically with 25 $\mu m^2$ area. The first trails of focusing light on SNSPDs were based on free space optics [20]. In these trails the device was placed in a cryogenic chamber with an optical window. This method suffers from few main disadvantages. The first limitation is that the optical window allows stray light reach the detector and
thus increases the dark counts (detection events not sourced by the experiment’s light source). The second limitation is the need for the complicated free space optic equipment. In particular there is a need for equipment for focusing the light on the small detector area and for active positioning, which is needed due to a detector-to-light-source alignment drift. More advanced light focusing methods were based on inserting an optical fiber into the cryogenic chamber and setting the detector close to the fiber’s tip. In some cases a positioning system was used to align the fiber to the detector [23, 24]. Such a system was installed in our lab, and can be seen in Fig. 6.2. The main limitation of such systems is, as with the free space optics case, the need to operate an active positioning during operation, to account for position drift. In addition, in order to focus the light on the sample, the fiber tip is positioned very close to it (<50 µm), a process which increases the risk of fiber-sample-contact and for scratching the thin superconducting layer. Other methods of focusing the light on the sample includes fixed positioning of the fiber to the sample [25, 26]. In these methods, the fiber is rigidly fixed on a sample holder, outside of the cryogenic chamber; optical or mechanical equipment can be used for fiber-to-sample-alignment. The sample and fiber are then cooled to the operation temperature. In this method the fiber-to-sample-alignment is usually limited. The best known example of rigid coupling is limited to few micrometers [27]. Another method, which we have tried, was to use a Fresnel lens which was patterned on the back side of the detector wafer, in order to focus the light on it [80]. A relatively good fiber-to-light-alignment has been achieved, but with the price of light lose due to the Fresnel lens multi foci.

The absorption efficiency is mainly limited by the fact that the superconducting layer is thin and thus mostly transparent. For example, for illumination on a 5 nm thick NbN SNSPD, with 50% fill-factor, about 30% of the light is reflected, 60% is transmitted, and only 10% is absorbed in the nanowire (see Sec. 6.2 for calculations). In order to increase the absorption efficiency, an optical cavity can be formed. An optical cavity will generally include at least two reflection surfaces. The light traveling in the cavity may then go back and forth between these two surfaces; provided that some conditions are met, constructive interference of the electromagnetic wave is created and the light is trapped for few cycles on average. In previous attempts [30], the detectors were put in a cavity for which the nanowire itself serves as one mirror and a second, metallic, mirror is added. These cavities has low finesse, as one of the mirrors forming it has high transmission coefficient.

6.2 Theory

This section provides the theoretical model of an on-fiber SNSPD. We first discuss the electrical model of the detector, and then the optical model of the fiber coupling to the detector and the optical cavity.
Figure 6.2: Cryogenic fiber to sample alignment system. (A) piezoelectric X-Y-Z motors, capable for sub micron positioning. (B) The fiber (marked with green line for clarity) is mounted on the motors. (C) a top plate on which samples are mounter. The plate is thermally coupled to the cryogenic environment.

Electrical Analysis

Hotspot Creation

Assume that we have a narrow section of a superconducting thin-film such that $d \ll w_b \ll l_b$, where $d$ is the film thickness, $w_b$ is the width of the section, and $l_b$ is the length of the section. Such a section is referred hereafter as a superconducting nanowire. The nanowire is cooled below its critical temperature and is at equilibrium state. At time $t = 0$, a photon with energy $\hbar \omega \gg 2 \Delta$ is absorbed into the nanowire, and creates a local non-equilibrium perturbation, namely a large number of normal electrons. Following [81, 82] the heat propagation in a two-dimensional thin film is governed by:

$$C d \frac{\partial T}{\partial t} = \kappa d \nabla^2 T + \alpha (T_0 - T), \quad (6.1)$$

where $\kappa$ is the thermal conductivity, $C$ is the heat capacitance of the electrons in the film per unit of volume, $\alpha$ is the heat transfer coefficient, and $T_0$ is the temperature of the coolant. Using the notations: $\tau_{th} = Cd/\alpha$, $\Lambda_D = \sqrt{\kappa d/\alpha}$ and $T_\Lambda = \hbar \omega / 4 \kappa d \tau_{th}$, we obtain the solution to Eq. (6.1):

$$T(r, t) = T_0 + T_\Lambda \frac{\tau_{th}}{t} \exp(-t/\tau_{th}) \exp \left[ -\frac{(r/\Lambda_D)^2}{4t/\tau_{th}} \right]. \quad (6.2)$$
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This solution can be understood as follows: shortly after impact, at a time \( t \lesssim \tau_{th} \), the temperature is increased by approximately \( T_\Lambda \), in a small region whose size is on the order of \( \Lambda_\rho \). The size of this region increases with time, as the temperature is decreased back to \( T_0 \). In the small area near the center of the hot-spot, where \( T(r) > T_C \) the film becomes normal conductive. Note that \( \tau_{th} \approx \tau_{el-ph} \), where \( \tau_{el-ph} \) is the interaction time between electron and substrate phonons [83].

In case where the size of the hotspot is smaller then the bridge’s width \( w_b \), hotspot formation by itself will not create a resistive blockade. To enhance the effect, the bridge is biased with a current \( I_b \) such that the current’s density \( j_b = I_b / (w \times d) \) is close to \( j_c \), the critical current density. When the hot-spot is formed, the supercurrent is routed toward the "side-walls" around it. As \( I_b \) is held constant, the current density in the side walls grows, and eventually exceeds \( j_c \). A resistive section, \( R_d \), is then formed on the bridge, and voltage drop is measurable.

Equivalent Circuit

We consider a simple equivalent circuit for our problem. The SNSPD meander is modeled using 3 lumped elements. First, the resistive part is modeled as a resistor \( (R_D) \) and an ideal switch, which are connected in parallel to each other. The resistor represents the normal section in the meander, created after a photon is absorbed.

Since a normal section exists only for a short period of time, the resistor is connected in parallel to an ideal switch. When the switch is closed, there is no normal section in the circuit. Second, the resistor and switch are connected in series to an inductor \( (L_k) \), which represents the meander superconducting kinetic inductance. The SNSPD is connected in parallel to a load resistor \( (R_L) \), whose role is to offload current from the detector after it is hot, and to allow switching back to the superconducting state. A constant current source sends bias current \( I_B \) to the circuit, and the voltage drop on the load resistor \( V_{out} \) is measured. The validity of this equivalent circuit is discussed by Annunziata et al. [29].

To this equivalent circuit we later add a capacitor in parallel \( C_m \), which represents capacitance between the leads of the SNSPD to the metallic mirror we add. One side of the mirror is capacitively coupled to the input lead, and the other side is coupled to the ground lead.

\[ R \parallel (R - L) \text{ Circuit} \]

Consider the circuit in Fig. 6.3. At time \( t < 0 \) the switch is closed. A steady state is reached when the currents are: \( I_{L_k} = I_B \) and \( I_{R_L} = I_{R_D} = 0 \). At time \( t = 0 \), a photon is absorbed in the SNSPD and the switch opens. The current starts routing toward the load resistor. Kirchhoff’s circuit laws reads:

\[ L_k \dot{I}_{L_k} + R_D I_{L_k} - R_L (I_B - I_{L_k}) = 0, \]  

(6.3)
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The current and voltage solution to Eq. (6.3) is:

\[ I_{L_k}(t) = \left[ \frac{R_L + R_D \exp(-t/\tau_{\text{up}})}{R_L + R_D} \right] I_B, \]  

(6.4)

and

\[ V_{\text{out}}(t) = [1 - \exp(-t/\tau_{\text{up}})] \frac{R_L R_D}{R_D + R_L} I_B, \]  

(6.5)

where \( \tau_{\text{up}} = L_k/(R_L + R_D) \). Thus, the current in the detector decreases exponentially. The current’s decrease is accompanied by a fast Joule heating of the \( R_D \). However, if \( \frac{R_L R_D}{R_D + R_L} I_B < I_c \) the steady state is a superconducting state, and after some time \( t_{\text{max}} \propto \tau_{\text{up}} \) the switch closes [29]. We assume that in that time \( I_{L_k} = \frac{R_L R_D}{R_D + R_L} I_B \approx 0 \) and \( I_{R_L} \approx I_B \). In this new state, the Kirchhoff circuit law reads:

\[ L_k \dot{I}_{L_k} - R_L (I_B - I_{L_k}) = 0 \]  

(6.6)

The solution to Eq. (6.6) is

\[ I_{L_k}(t + t_{\text{max}}) = [1 - \exp(-t/\tau_{\text{down}})] I_B, \]  

(6.7)

and

\[ V_{\text{out}}(t + t_{\text{max}}) = \exp(-t/\tau_{\text{down}}) R_L I_B, \]  

(6.8)

where \( \tau_{\text{down}} = L_k/R_L \). We note that \( R_L \ll R_D \). Thus, the maximal pulse height is \( V_{\text{max}} = \frac{R_L R_D}{R_D + R_L} I_B \approx I_B R_L \approx I_c R_L \), and the pulse width is proportional to \( \tau_{\text{up}} + \tau_{\text{down}} \approx \tau_{\text{down}} = L_k/R_L \).

Figure 6.3: SNSPD schematic electrical circuit. \( R_D \) is the device’s normal resistance, \( R_L \) is the load resistance, \( L_k \) is the kinetic inductance.
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Figure 6.4: SNSPD in a cavity schematic electrical circuit. $R_D$ is the device’s normal resistance, $R_L$ is the load resistance, $L_k$ is the kinetic inductance, and $C_m$ is the mirror capacitance.

**R||C||(R-L) Circuit** Next, we add a capacitor to the circuit, and consider the circuit in Fig. 6.4. Again, at time $t < 0$ the switch is closed. A steady state is reached when the currents are: $I_{Lk} = I_B$, $I_{RL} = I_{RD} = I_{Cm} = 0$.

At time $t = 0$, the switch opens due to a photon-absorption event. The current starts routing toward the load resistor and the capacitor. Kirchhoff’s circuit law reads:

$$L_k \dot{I}_{Lk} + R_D I_{Lk} = R_L I_{RL} = V_{Cm} = V_{out}, \quad (6.9)$$

and

$$I_{Lk} + I_{RL} + C_m \dot{V}_{Cm} = I_B. \quad (6.10)$$

Replacing Eq. (6.9) in Eq. (6.10) we get:

$$C_m L_k \dot{I}_{Lk} + \left[ C_m R_D + \frac{L_k}{R_L} \right] \dot{I}_{Lk} + \left[ 1 + \frac{R_D}{R_L} \right] I_{Lk} - I_B = 0. \quad (6.11)$$

This is a second order ordinary differential equation, which can be solved having the initial conditions: $I_{Lk} = I_B$ and $\dot{I}_{Lk} = 0$. The roots of the characteristic equation of Eq. (6.11) are:

$$\tau_{1,2} = \frac{1}{2} \tau_{up} \left[ 1 + \frac{C_m R_D R_L}{L_k} \pm \sqrt{\left( 1 - \frac{C_m R_D R_L}{L_k} \right)^2 - \frac{4 C_m R_L^2}{L_k}} \right]. \quad (6.12)$$

We will require the mirror capacitance to be small such that $C_m \ll L_k/(R_L R_D)$. Under this requirement we get an overdamped 2nd order linear ordinary differential equation, in which case the two roots are real and the condition $\tau_2 \ll \tau_1 \approx \tau_{up}$ holds.
The evolution dynamic is dominated by the larger root \( \tau_1 \). The solution can now be approximated by the solution of Eq. (6.3).

The current in the detector decreases exponentially, and again, after time \( t_{\text{max}} \) the switch closes. We assume that in that time \( I_{L_k} = I_{Cm} = 0 \) and \( I_{R_l} = I_B \). Now, the Kirchhoff’s circuit law reads:

\[
C_mL_k\ddot{I}_{L_k} + \frac{L_k}{R_l}\dot{I}_{L_k} + I_{L_k} - I_B = 0.
\]

Again we have a second order ordinary differential equation, with roots of the characteristic equation:

\[
\tau_{3,4} = \frac{L_k \pm \sqrt{L_k^2 - 4C_mL_kR_l^2}}{2R_l} = \frac{1}{2} \tau_{\text{down}} \left[ 1 \pm \sqrt{1 - \frac{4C_mR_l^2}{L_k}} \right].
\]

The condition \( C_m \ll L_k/(R_lR_B) \) is sufficient to assure an overlapped state, in which case the dynamic is determined by \( \tau_3 \approx \tau_{\text{down}} \), and the solution is similar to that of Eq. (6.6).

We conclude that by keeping the mirror capacitance much smaller than the calculated limit, the behavior of the detector will remain as in the case without the mirror.

### Optical Analysis

**Mode in an Optical Fiber**

Consider an optical fiber, that has a core with refraction index \( n_1 \) and diameter \( D_{\text{core}} \), and a clad with refraction index \( n_2 \), so that \( n_1 > n_2 \) and \( \Delta n = n_1 - n_2 \ll n_{1,2} \).

Consider an electromagnetic wave, with frequency \( \omega \), free space wave length \( \lambda = 2\pi c/\omega \), and free space wave number \( k_0 = \omega/c \), propagating in such a fiber. An important parameter in this fiber is a dimensionless cutoff condition [84]:

\[
V = \frac{1}{2} k_0 D_{\text{core}} \sqrt{n_1^2 - n_2^2} \approx \frac{1}{\sqrt{2}} k_0 D_{\text{core}} \sqrt{n_1 \Delta n}.
\]

For a fiber having \( V < 2.405 \), there is only one mode supported by the fiber. The electrical field distribution in the \( z \)-direction in such a condition can be well-approximated by [84]:

\[
E_z(z,r) = E_0 \exp(-4r^2/D_0^2) \exp(i\beta z)
\]

where \( D_0/D_{\text{core}} \approx 0.65 + 1.619V^{-3/2} + 2.879V^{-6} \) is the waist, and \( \beta \) is the propagation constant in the material which can be calculated using the relation

\[
\beta / k_0 = n_{\text{eff}} = n_2 + b(n_1 - n_2),
\]

with \( b(V) \approx (1.1428 - 0.9960/V)^2 \).
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Transfer Matrix Formulation

Consider a light mode, traveling in the fiber, that encounters a general diffuser located between \( z = 0 \) and \( z = d \geq 0 \). The diffuser can couple the traveling wave with the returning wave and change amplitudes and/or phases (see Fig. 6.5). We consider a description of such a diffuser in the following formalism [85]: The waves outside the diffuser are described by the equations:

\[
E_z(z, r) = \exp\left(-4r^2/D_0^2\right) \begin{cases} 
E^+_t \exp(i\beta_t z) + E^-_t \exp(-i\beta_t z) & z < 0 \\
E^+_r \exp(i\beta_r z) + E^-_r \exp(-i\beta_r z) & d < z
\end{cases}
\]  

(6.18)

where \( E^\pm_t, E^\pm_r \) are constant amplitudes. The diffuser will be described by the general matrix:

\[
\begin{pmatrix} E^+_r \\ E^-_r \end{pmatrix} = M \begin{pmatrix} E^+_t \\ E^-_t \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} E^+_t \\ E^-_t \end{pmatrix}
\] 

(6.19)

The reflection and transfer coefficients can be calculated from the transfer matrix using [85]: \( t = 1/m_{11}, \ r = m_{21}/m_{11} \).

When two diffusers with transfer matrices \( M_1, M_2 \) are present and arranged in series, we can calculate the equivalent transfer matrix \( M_{eq} = M_1 M_2 \).

In the following subsections we intend to analyze the various optical components in our system, using the transfer matrix formalism.

Light Traveling in a Section of a Fiber

Light traveling in a section of a fiber with length \( d \), does not face any diffusion. As so, there is no interaction between the forward and returning waves. There is, however,
phase accumulation in this fiber section. The transfer matrix for this fiber is thus:

\[ M_d = \begin{pmatrix} \exp(-i\beta d) & 0 \\ 0 & \exp(+i\beta d) \end{pmatrix} \].

(6.20)

**Metallic Mirror**

We now consider the light mode traveling in an optical fiber and impinging upon a silver metallic mirror. We define \( n_{Ag} = n(1 + ik) \) as the refractive index of silver and \( n_{SiO_2} \) as the refractive index of glass. The transfer matrix for the fiber-to-silver interface is:

\[ M_{SiO_2\rightarrow Ag} = \frac{1}{2n_{Ag}} \begin{pmatrix} n_{Ag} + n_{SiO_2} & n_{Ag} - n_{SiO_2} \\ n_{Ag} - n_{SiO_2} & n_{Ag} + n_{SiO_2} \end{pmatrix} \].

(6.21)

The light then continues to propagate in the silver layer, with a propagation constant \([86] \beta_{Ag} = 2\pi(n_{Ag})/\lambda\). The transfer matrix for a \( d_{Ag} \)-thick silver layer is thus:

\[ M_{Ag} = \begin{pmatrix} \exp(i\beta_{Ag} d_{Ag}) & 0 \\ 0 & \exp(i\beta_{Ag} d_{Ag}) \end{pmatrix} \].

(6.22)

The light which passes through the silver layer faces a silver-to-air interface, with a transfer matrix of:

\[ M_{Ag\rightarrow Air} = \frac{1}{2} \begin{pmatrix} 1 + n_{Ag} & 1 - n_{Ag} \\ 1 - n_{Ag} & 1 + n_{Ag} \end{pmatrix} \].

(6.23)

The total transfer matrix of the mirror is \( M_M = M_{SiO_2\rightarrow Ag} M_{Ag} M_{Ag\rightarrow Air} \). Fig. 6.6 shows the reflection, transmission and absorption amplitudes, as a function of film thickness, for \( \lambda = 1550\)nm. For \( d_{Ag} \rightarrow \infty \) we get the reflection probability \(|r|^2 = 0.975\). Similar calculations can be done for the NbN detector layer, replacing \( n_{Ag} \) and \( d_{Ag} \) with \( n_{NbN} \) and \( d_{NbN} \).

**Fiber Bragg Grating**

Consider a grating of the refraction index of the fiber, having the following simple form \([88]\):

\[ n(x, y, z) = n_t(x, y) \left\{ n_{DC} + \nu \cos \left( \frac{2\pi n_{eff} z}{\lambda_0} \right) \right\}, \]

(6.24)

where \( \lambda_0 \) is the grating period, \( n_t \), \( n_{DC} \) and \( \nu \) are dimensionless, \( n_t \times n_{DC} = n_{eff} \) and \( n_t \) is such that:

\[ \int \int dxdyE_0 \exp(-4r^2/D_0^2)n_t(x, y) = 1. \]

(6.25)

This grating is communally referred as "Fiber Bragg Grating" (FBG). We introduce the following parameters: \( \kappa = \frac{\pi}{\lambda} \nu \) is the grating strength, \( K_0 = \frac{2\pi n_{eff}}{\lambda_0} \) is the
6.2. THEORY

Figure 6.6: (Color) The reflection, transmission and losses in a silver metallic mirror as a function of mirror thickness. The left-axis values are of the amplitudes (square absolute values) of transmission ($|t|^2$, solid blue), reflection ($|r|^2$, solid red), power conserved ($|t|^2 + |r|^2$, solid black), and losses ($1 - |t|^2 - |r|^2$, solid green). The right-axis values are of the phase shift of transmission ($\angle t$, dashed blue) and reflection ($\angle r$, dashed red). We have used the refractive index of glass, $n_{\text{SiO}_2} = 1.5$ [87], and the refractive index of silver, $n_{\text{Ag}} = 0.514 - 10.8i$ [87].

grating momentum, $\beta = \frac{n_{\text{eff}}}{\lambda}$ is the wave momentum, $\Delta K = 2\beta - K_g$ is the momentum difference and $\gamma_B = \sqrt{\kappa^2 - \Delta K^2}$. The transfer matrix of the FBG of length $\Lambda$ is given by [88]

$$M_B = \begin{pmatrix}
\cosh(\gamma_B \Lambda) - i\frac{\Delta K}{\gamma_B} \sinh(\gamma_B \Lambda) & -i\frac{\kappa}{\gamma_B} \sinh(\gamma_B \Lambda) \\
\frac{\kappa}{\gamma_B} \sinh(\gamma_B \Lambda) & \cosh(\gamma_B \Lambda) + i\frac{\Delta K}{\gamma_B} \sinh(\gamma_B \Lambda)
\end{pmatrix}. \quad (6.26)$$

The reflection coefficient of the grating can be calculated as:

$$r = \frac{M_{21}}{M_{11}} = -\frac{\kappa \sinh(\gamma_B \Lambda)}{\Delta K \sinh(\gamma_B \Lambda) + i\gamma_B \cosh(\gamma_B \Lambda)}, \quad (6.27)$$

and the reflection probability is:

$$|r|^2 = \frac{\sinh^2(\gamma_B \Lambda)}{\cosh^2(\gamma_B \Lambda) - \frac{\Delta K^2}{\kappa^2}}, \quad (6.28)$$

for $\Lambda \gg \lambda_0$, and for real $\gamma_B$. The condition on the wave length is then $|\Delta \lambda| < \frac{1}{2} \nu \lambda_0$. For grating with $\lambda_0 = 1550$ nm and $\Delta \lambda = 0.3$ nm we get $\nu = 2.9 \times 10^{-4}$. In Fig. 6.7 we plot few estimations to the reflection constant around $\lambda_0 = 1550$ nm.
Figure 6.7: (Color) The reflection amplitude off FBG with $\lambda_0 = 1550$ nm. (A) $\lambda$ vs. $|r|^2$ for several $\Lambda$ values, with $\nu = 10^{-4}$. (B) $\lambda$ vs. $|r|^2$ for several $\nu$ values with $\Lambda = 2$ cm.

Figure 6.8: A scheme of the on-fiber SNSPD in an optical cavity. The cavity is composed of a Bragg mirror, an SNSPD, and a silver mirror.

Cavity Formation

When installing several optical components in series, an optical cavity can be formed, i.e., a section of the fiber for which light travels back and forth in the cavity, and due to constructive interference only a small portion of the light reflects off the cavity (see Fig 6.8). A cavity is said to be critically coupled when the reflection component is zero. Fig. 6.9 shows the reflection and absorption coefficients of an optical cavity designed with critical coupling. When the absorption coefficient is 1, all the light is absorbed in the detector or the silver mirror. This is the optimal possible case; all the light enters the cavity and none is returned. In a single run, the absorption in the silver mirror is less than 0.5% and the absorption in the detector is about 10%; therefore, most of the light is absorbed in the detector.
6.3. FABRICATION

![Graph showing Reflection/Absorption vs wavelength](image)

Figure 6.9: (Color) Reflection (red) and absorption (green) coefficients of an optical cavity as a function of light wavelength. The parameters for this cavity are: $n_{Ag} = 0.514+20.8i$, $n_{NbN} = 5.23+5.82i$ with fill factor of 0.3, FBG length $L = 0.32$ cm, cavity length $d_{SiO2} = 1.2$ cm. Critical coupling is archived for wavelengths of $\lambda = 1549.893$, 1549.955, 1550.017, and 1550.080 nm.

Fiber-to-Detector Alignment

In order to estimate the effect of the center-to-center misalignment on detection efficiency, we calculate $\eta_c$, namely the overlap between the Gaussian optical mode in an optical fiber [89] and a detector located close to the top facet of the fiber:

$$
\eta_c = 4(\pi D_0^2)^{-1} \iint \exp(-4r^2/D_0^2) \times g(r) dA,
$$

(6.29)

where $D_0$ is the the mode field diameter and $g(r)$ takes a value of 1 if $r$ is in the area of the detector, and vanishes otherwise. In Fig. 6.10 we plot $\eta_c$ for a several detectors, with several $x_{CC}$ values, where we assume a detector area fill-factor of 100% and an SMF-28 fiber. We can see that for $x_{CC} \geq 10$ $\mu$m and relatively large detector with diameter of 15 $\mu$m (panel a), $\eta_c$ is less than 20%. Even for $x_{CC} = 5$ $\mu$m (panel b), $\eta_c$ is limited by 50% for detectors with less than a 10-um-diameter. Note that for shorter wavelength fibers, the mode field diameter is even smaller and consequently misalignment results in even larger light loss.

6.3 Fabrication

Complete Process Overview

The complete fabrication process was done on the top facet of a zirconia ferrule, taken from a standard flat polished fiber connector (FC-UPC), that holds a single mode fiber for the telecommunication bandwidth (Corning SMF-28 [90]). The ferrule facet has
Figure 6.10: The coupling efficiency ($\eta_C$) between the Gaussian light beam with diameter $D_0$ and the detector with size $D_D$, where the center-to-center misalignment is $x_{CC}$. We assume $D_0 = 10.4 \mu m$ (SMF-28) and a detector fill factor of 1. (a) $\eta_C$ vs. $x_{CC}$ for circular detectors with $D_D = 5, 10$ and $15 \mu m$ in solid blue, dashed green and dashed-dotted red, respectively. (b) $\eta_C$ vs. $D_D$. In solid blue (dotted blue) $x_{CC} = 0$ for a circular (rectangular) detector with diameter (edge) of $D_D$. In dashed green, $x_{CC} = 5 \mu m$. In dashed-dotted red: the light loss due to $x_{CC} = 5 \mu m$ relative to the $x_{CC} = 0$ case, i.e., the solid blue curve divided by the dashed green curve.

Figure 6.11: (Color) A scheme of the on-fiber SNSPD fabrication steps. (A) A fiber with FBG mirror section and a zirconia ferrule are used. (B) The fiber is glued into a hole in the ferrule and polished. (C) Contact pads are deposited through a mechanical mask. (D) An NbN layer is deposited through a mechanical mask. (E) The SNSPD is patterned in two FIB steps. (F) SiO$_2$ is deposited over the detector. (G) A silver mirror is deposited over the detector to form a cavity.
a diameter of 2 mm; the fiber with a 125-um-diameter is epoxy-glued concentrically with the ferrule. We evaporated 5-nm-thick chromium film followed by a 200-nm-thick gold film through a mechanical mask to form bonding pads. Next, a 10-nm-thick niobium nitride (NbN) film was deposited using a DC-magnetron sputtering system. The sputtering process was executed from a niobium target in a vacuum chamber filled with a mixture of argon and nitrogen gasses while keeping the sample at room temperature [91, 92]. The NbN film was covered in-situ with 100 nm of aluminum.

To pattern the detector, we first narrowed the NbN film to a 25-um-wide bridge using a focused ion beam (FIB) system with a relatively high current (2.1 nA). During this step the aluminum layer protected the NbN film from an exposure to the ion beam, and reduces gallium poisoning [35, 34, 93]. After the first lithography step, the aluminum layer was wet-etched. The sample then underwent a second lithography step, in which a meander was formed using low-current (9.7 pA) FIB patterning. A fabricated device can be seen in Fig. 6.12.

To add a cavity, we used FBG mirror. The FBG was purchased from O/E Land Inc. [94] with a 0.3-nm reflection band around 1550-nm wavelength. In this case we used a FuseLite fiber connectors by Corning, which allowed for a short fusing distance (1 cm) from the connector facet, and thus enable a short optical cavity. A last deposition step was added where a $SiO_2$ buffer and a top mirror were deposited over the SNSPD.
Figure 6.13: (Color) A ferrule holder used for deposition of metals on the fibers. The top (left) and bottom (right) faces of the holder are seen. On the top face, there are places for three mechanical masks. On the bottom side, there is a compartment which holds the fibers during the deposition process.

**Thin Film Deposition**

**Sample Holder and Mechanical Masks**

A special sample holder was manufactured in the mechanical shop for all the deposition processes (Fig 6.13). This sample holder can hold 3 fiber ferrules. The holder has a back compartment, in which a few tens of centimeter of a fiber can be wound and stored, keeping them protected from the materials being deposited. On the top facet of the holder, circular mechanical masks can be placed in contact with the ferrule’s top facet. The masks are made from 0.1-mm-thick stainless steel sheet, and are defined using laser cutting as a circle with a diameter slightly smaller than 9 mm, and the desired evaporation feature in the middle. The circular shaped mask fits a circular hole in the sample holder, and is concentric with the fiber connector and the fiber itself. Due to the precision of mask fabrication and mask alignment, the final pattern on the sample has an alignment-precision tolerance of about 30µm with minimal feature size of about 80µm. Due to shadows during evaporation and sputtering, the pattern on the mask tend to have features that are a few micrometers larger than those of the mask, with a gradual profile. The masks are placed over the sample, so that their angular position cannot be determined. As we used two or more masks in the fabrication procedure, we designed all masks to be angular-rotation invariant, i.e., the deposition pattern is limited to be a circular hole in the middle of the mask. In each procedure there was one mask which did not have angular rotation symmetry. This mask defined the direction of the sample, and was usually used to define the contact pads.
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Table 6.1: Sputtering parameters for NbN layers. $I_{\text{dis}}$ is the discharge current, $P_{\text{Ar}}$ ($P_{\text{Tot}}$) is the argon partial (total) pressure in the vacuum chamber, MFC$_{\text{Ar}}$ (MFC$_{\text{N}}$) is the argon (nitrogen) mass flow controller parameter (with arbitrary units form 0 to 2047), TV is the throttle valve closure position (0% is open, 100% is fully closed), $t$ is the layer thickness as measured by the crystal monitor in the sputtering chamber, $R_{\text{C}}$ is the room square resistance, $T_C$ is the critical temperature.

**NbN Sputtering**

The deposition of high quality NbN films was a key issue in the overall fabrication process. Most of the SNSPDs presented in papers are deposited on sapphire or MgO substrates which are heated to about 800°C during the sputtering process. Sapphire and MgO are lattices with good matching to the NbN lattice; this helps to create a highly ordered NbN lattice starting from the first few layers of material. Heating the substrate helps annealing the deposited material into an ordered lattice.

In our devices, the substrates were polished amorphous glass fibers. Existence of epoxy glue and plastic fiber coating materials made it impossible to heat the stage, since these materials might have evaporated and contaminated the sputtering machine’s vacuum chamber. Our thin NbN films were deposited using a DC-magnetron sputtering system from an Nb target with partial pressure of argon and nitrogen gases [91, 92]. The substrate fibers were kept at room temperature using water cooling. The sputtering parameters for NbN layers are listed in table 6.1. After many trails and errors, we were able to produce thin NbN layers with $T_c$ of 13 K and above.
Figure 6.14: An optical image of a detector, obtained using IR sensitive camera. The nanowire is not visible, due to optical resolution limitations (see Fig. 6.12b for higher resolution). (a) The device under external illumination. (b) The external illumination is lowered and light is injected from the fiber, behind the detector (see Fig. 6.12c). The high accuracy alignment of the light spot to the detector is demonstrated; the center of the Gaussian light beam overlaps the area of the detector.

FIB Lithography

The FIB imaging system, used to align the detector with the fiber center, allows achieving $x_{CC}$ of less than 1 μm, and is limited mainly by the fiber core-to-clad misalignment that is given to be $< 0.5$ μm by Corning [90]. Fig. 6.14 demonstrates the low center-to-center-misalignment in our devices.

The usage of FIB for SNSPD is not common in the fabrication processes of SNSPD. In fact all the SNSPDs reported to date in literature, were fabricated using electron beam lithography (EBL) with combination of dry etching (electron cyclotron resonance (ECR) or reactive ion etching (RIE)) machines [21, 95]. EBL requires e-beam resist deposition on the surface of the substrate, which is usually done by spin coating. Since the ferrule’s facet has a small diameter, and since it is hard to spin a long fiber around its center, we have decided not to use EBL. Some attempts with EBL are described in the appendix 6.A.

In order to validate the quality of the FIB-fabricated meanders, intended to serve as SNSPDs, a series of test samples were made on silicon covered by an amorphous SiN substrate. A summary of the parameters and the data taken from these samples is provided in table 6.2. The samples were measured for their critical current at 4.2 K and their resistance at room temperature. The critical current is a property which measures the most constricted spot in the meander; as the current is equal over the length of the wire, current density will cross the critical value first in the narrowest spot. Thus, keeping $j_c$ close to the current density of a non patterned sample suggest no constriction exist in the meander, i.e., the width of the meander is even. On the other hand, room temperature resistivity is an accumulative quantity; it is an integral of the specific resistivity over the length of meander. Thus, having an average specific resistivity similar to that of a non patterned sample suggests that there are no short-circuits between the meander lines, meaning that the FIB dwelling was sufficient.

In addition, V-I curves (current as a function of applied voltage) and I-V curves
Table 6.2: Experimental data obtained from several FIB-fabricated NbN meanders that make up SNSPDs. The parameters in the titles are: \( t, w, \) and \( l \) are the meander’s thickness, width, and length respectively, \( d \) is the FIB dwell depth with Si program and 9.7-pA current, \( I_c(j_c) \) is the superconducting critical current (density) at 4.2 K, \( R(\rho) \) is the normal temperature (specific) resistance after \( I_c \) was crossed. The meander descriptions: "No FIB" is a wire which was formed using photolithography and RIE etching. "Meander \( n \times m \)" is a meander which cover an area of \( n \) by \( m \) square micrometers. "line \( n \)" is a single wire \( n \) micrometers long.

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{Description} & t[\text{nm}] & w[\text{nm}] & l[\mu\text{A}] & d[\text{nm}] & I_c[\mu\text{A}] & j_c[\text{MA/cm}^2] & R[\Omega] & \rho[\mu\Omega\text{cm}] \\
\hline
\text{No FIB} & 10 & 7300 & 40 & - & 2000 & 2.74 & 1500 & 274 \\
\text{Line 1} & 10 & 100 & 1 & 30 & 10.7 & 1.07 & 3100 & 310 \\
\text{Meander} 2 \times 2 & 10 & 160 & 20 & 30 & 22.6 & 1.41 & 33000 & 264 \\
\text{Meander} 2 \times 2 & 10 & 160 & 20 & 20 & 34.7 & 2.72 & 34000 & 272 \\
\text{Meander} 5 \times 5 & 10 & 160 & 140 & 30 & 35.0 & 2.19 & 180000 & 206 \\
\text{Meander} 5 \times 5 & 10 & 160 & 140 & 40 & 37.0 & 2.31 & 205000 & 234 \\
\text{Meander} 5 \times 5 & 8 & 160 & 140 & 40 & 21.8 & 1.70 & 310000 & 283 \\
\hline
\end{array}
\]

(voltage as a function of applied current) were taken for each sample. The use of V-I and I-V curves makes it possible to estimate the critical current for various constrictions in the meander. Fig. 6.15(A) shows an example of V-I curves. The voltage is raised from 0; as some of it falls on the normal conducting wires leading to the sample, a finite current which is proportional to the voltage is created in the sample (Ohm’s law). As the current crosses the critical current in the meander’s most constricted spot, this spot becomes normal conductive. The total resistivity increases and the current decreases. The constricted area stays normal conductive due to Joule self-heating. The voltage continues to increase until the current crosses the critical current in the next constricted spot. This spot now becomes normal, the current drops and the process continues. Fig. 6.15(B) shows an example of I-V curves. In this case the presence of constrictions in the wire causes the curve to have a "stairs" shape, i.e., whenever a spot becomes normal conductive the resistivity increases and the voltage "jumps". Therefore, non-constricted meanders have a continuous I-V curve.

In our research we found little documentation for the characterization of SNSDPS using V-I and I-V curves. We assume that the reason is that SNSPDs are operated when shorted by 50 – \( \Omega \) load resistors (see Sec. 6.4). In this mode of operation, upon the first constriction becoming normal, the measured current is equal to \( V_{in}/R_L \), where \( V_{in} \) is the source voltage and \( R_L \) is the load resistor. For further change in the phase of the nanowire there will not be any change in the output signal. Thus, no information about the nanowire uniformity can be achieved from such a measurement.
Figure 6.15: (Color) A V-I curve and an I-V (B) curve of a DC-biased meander. The solid blue (dashed red) curve represent the raising (falling) voltage or current respectively.

6.4 Single Photon Detection Experimental Results

To characterize the detection performance, the fabricated device was inserted into a compact holder (Fig. 6.12c), and was wire-bonded to an SMA connector. The SMA was used for both DC-current bias input and fast pulses output. The experimental setup is schematically presented in Fig. 6.16. We used an attenuated monochromatic laser at vacuum wavelength $\lambda_0 = 1550$ nm as our source. The laser source is assumed to output photons at a Poisson distribution, $P(k) = \Xi^k e^{-\Xi}/k!$, where the average emission rate in the time interval $\tau$ is characterized by the nominal output power:

$$\Xi_{\tau} = \frac{P_{\text{laser}} \tau}{hc/\lambda_0};$$

with $P_{\text{laser}}$ is the laser power source, $h$ is the Planck constant, and $c$ is the speed of light in vacuum.

Fig. 6.17, shows the experimental results of the system detection efficiency (noted as $\eta_{SDE}$). Inset (A) shows the average photon-detection rate for an average photon flux rate, for a few values of $I_B$. As observed, the ratio is independent of the flux rate, and is defined as $\eta_{SDE}$. Inset (B) shows $\eta_{SDE}$ as a function of the bias current in one of our devices. The critical current in the presented device is $I_c = 42 \mu$A. The dark count, when the device is biased just below $I_c$, is measured to be 10 Hz. Since $\eta_{SDE}$ does not saturate when approaching $I_c$, and since the dark count rate is low compared to other devices reported in literature [21], we conclude that the level of the
6.5. SUMMARY AND FUTURE WORK

critical-current uniformity along the meander is relatively low in our device. Further work is needed to identify the underlying mechanisms that are responsible for the poor uniformity.

In order to ensure that the system detects photons arriving at the Poisson distribution, we measured the statistics of detection events. First, we measured the distribution of detection events in time intervals of 1 μs. The distribution fitted well the Poisson distribution (Fig. 6.18A). In addition we measured the statistics of time intervals between sequential detection events. We started an experiment at some arbitrary time $t = 0$ and measure time of arrival of first photon. We defined $P(D_{τ_0})$ as the probability for the case were there are no detection events in the time interval $0 < τ < τ_0$ and there is a detection event in the time interval $τ_0 < t < τ_0 + δτ$ of some small $δτ$. For a memoryless process we expected to find that for any $τ_0 < τ_1$

$$ P(D_{τ_1} | τ_1 > τ_0) = P(D_{τ_1 - τ_0}) = \exp (-η_{SDE} Ξ_{τ_1 - τ_0}). \quad (6.31) $$

Fig. 6.18 shows an experiment measuring $P(D_t)$. Time traces of the system’s output were divided into bins of 2.5 ns, and an histogram of the differences between sequential detection events was calculated. For $τ < 15$ ns we see that the system has a dead time, with very few detection events; the time duration of a pulse is 20 ns, and we expect no detections at this period. For times $τ > 30$ ns we see that the distribution fits well an exponential distribution, with decay time of $τ = 1 \mu s/(η_{SDE} Ξ_{τ_0})$. For the time period of $15$ ns $< τ < 30$ ns we see a slight increase in detection efficiency, with $P(D_{τ=30 \text{ ns}}) = 0.15$. For the fitted exponential distribution, $P(D_{τ=30 \text{ ns}}) = 0.1$.

6.5 Summary and Future Work

In summary, we introduced and analyzed a novel configuration for an SNSPD on the tip of a fiber and inside an optical resonator. We fabricated and measured an on-fiber SNSPD, with very low misalignment between the incoming light and the detector area. As we were able to integrate the detector on the tip of the fiber, we measured the true system detection efficiency with a simple experimental setup, that did not include any additional optics or alignment equipment. Though we achieve high coupling efficiency, more research is needed in order to increase the system’s detection efficiency. The integration with the cavity has so far failed experimentally, mainly because of difficulties in achieving FBG-integrated fibers with smooth tip to serve as a substrate. Future work may concentrate on improving surface conditions, by either using a better polish techniques or adding buffer layers between the fiber and the detector.
Figure 6.16: The experimental setup. The device is connected in parallel to a load resistor, \( R_L \), to prevent latching [29]. We use CW monochromatic laser at 1550 nm as a light source. The light is attenuated at room temperature to 10 nW and sent into a cryostat where the device is cooled to 3.5 K. Current is sourced from a computer-controlled battery. The current enters the direct current (DC) port of a bias-T and into the detector. The output signal is sent from the RF port of the bias-T through a cryogenic amplifier and up to room temperature using a semirigid coax cable. The signal is further amplified at room temperature and sent to a pulse counter (Stanford Research module SR-400).

Figure 6.17: (Color) Experimental data from our single-photon detector. (A) Pulse count vs. photon flux, for several input currents, showing that detection efficiency is not dependent on the photon flux. The error-bars represent the standard deviation of the measurement. (B) The system detection efficiency \( \eta_{SDE} \) as a function of the bias current. The error-bars represent the standard deviation for the measurement. (C) An electrical pulse from the detector after a single-photon detection event.
Figure 6.18: (Color) (A) Probability of detection events in time intervals of 1 µs (blue bars), and the calculated Poisson distribution with fit parameter of $n_{SDE} \times \Xi_{1\mu s} = 3.9$ events (B) The time interval between two sequential single-photon detection events probability density: Experimental data (solid blue) and a fit to exponential decay with $\tau = 1 \mu s / (n_{SDE} \times \Xi_{1\mu s}) = 253 \text{ns}$ (dashed red). Orange coloring is artificial to emphasize deviation from the exponential time decay for $t < 30 \text{ ns}$. (C) A snapshot showing single-photon detection event pulses as a function of time. Thirty-two detection events are recorded in the randomly selected time period of 10 µs seen in the graph.

6.A Step-by-Step Fabrication Recipe

On-Fiber SNSPD Fabrication

The following procedure was used for the fabrication of the on-fiber SNSPD. Step 11 hereafter is optional; it was successfully tried, on the route to a full cavity SNSPD, but without integration of the detector with a FBG mirror, it is not necessary for a working device.

1. Start with a UPC fiber connector. The connectors are made from the following parts:

   a) A Zirconia (ZrO$_2$) ferrule, 2.5 mm in diameter, 20-mm-long.

   b) A single mode fiber designed for 1550-nm-wavelength, 125 µm in diameter. The fiber is additionally coated with diameter plastic shield with a diameter of 250 or 900 µm.

   c) About 4 cm of the plastic shield is removed from the fiber tip using a commercially purchased dedicated instrument.

   d) The fiber is cleaved to 21-mm-long tip above the plastic cover.

   e) The fiber is epoxy glued to a hole in the middle of the ferrule, and polished to the surface of the ferrule height.
These steps were repeated in our lab with partial success. The yield was on
the order of 50%, mostly due to fiber breakage, and the fiber facet were not as
smooth as in commercially acquired fibers.

2. Clean the fiber:

a) Thoroughly clean the ferrule in solvents: acetone, methanol, and iso-
propanol. Clean the ferrules in an ultrasonic bath for at least 5 minutes
per solvent and then dry them using a nitrogen gun to remove excess iso-
propanol. During this step, acetone should not touch the plastic shielding
of the fiber as it would dissolve it.

b) Rub the fiber’s plastic shield with a paper cloth soaked in isopropanol.

3. Place the sample on the sample holder. Special sample holders were manufac-
tured for this purpose (see section 6.3).

4. Cover the sample with a mask for the contact layer deposition. The deposition
masks are laser-cut from a 50μm thick stainless steel sheet. This mask type
has two 1 mm² square holes, spaced from each other by 90μm. Place the mask
in such a way that it covers the core but leaves the sides of the fiber clear for
evaporation. Use an optical microscope for the placement.

5. Evaporate thermally 5 nm of chromium followed by 200 nm of gold to be used
as contacts.

6. Replace the mechanical mask for contact-pads deposition to a mask for NbN
deposition. This mask type has a round hole in the center with a diameter of
100 – 200μm. Use an optical microscope to make sure that the core, and the
two contact pads are clear for deposition.

7. Sputter an NbN layer. This is the most critical step in the process. More details
are given in section 6.3

a) Sputter 10 – 15 nm of NbN. Typical parameters are given in 6.3.

b) In-situ, sputter 100 nm of aluminum.

8. First focused ion beam lithography (FIB) step. Narrow the NbN and Al layers
to a 20μm by 100μm section, covering the fiber core. In addition, pattern
alignment marks 10μm from the fiber center. Use high magnification so that
the entire fiber is seen in a single FIB machine field. Use the fiber border to
align. Set the FIB current to 2.1 nA. The aluminum layer protects the NbN
form gallium poisoning, so there is no need to worry about the sample’s exposure
to the ion beam.
9. Etch the aluminum layer using a TMAH solution for 1 min. Use an optical microscope to make sure that the entire layer was removed. Use isopropanol to stop the etching. Dry the sample using a high-pressure air or nitrogen gun.

10. Second focused ion beam lithography (FIB) step. Pattern the microbridge. Set magnification to a field of view of the size of the meander. Set the FIB current to 9.7 pA. Use Electron beam to find your position. At no point critical NbN layer (an area that will be used as superconductor) should be exposed to the ion beam. Use small steps to center the ion beam on the alignment mark. Travel to the core of the fiber and write the meander without exposing the field (i.e. using "Play" mode in the FIB window).

11. Optional steps: adding a metallic mirror.
   a) Place the sample on the deposition sample holder (see section 6.3).
   b) Cover the sample with a mask for SiO₂ layer deposition. This mask type has round hole in the center, with diameter of 0.2 mm. Use an optical microscope to make sure the entire fiber is covered.
   c) Thermally evaporate 100 nm of SiO₂.
   d) Replace the mechanical mask with a mask for mirror layer deposition. This mask type has a round hole in the center, with diameter of 0.1 mm. The hole is smaller than the hall of SiO₂ mask, so that short-circuits will not be created. Use an optical microscope to make sure the fiber core is covered, and that no gold or NbN is exposed from the SiO₂.
   e) Thermally evaporate 200 nm of silver, gold or aluminum.

12. Insert the ferrule into the sample measurement holder containing an SMA connector. Bond one gold pad to the SMA signal line and the other pad to the ground.

13. Fuse the sample’s fiber to a longer fiber inside a cryostat.

Other Processes

The previous sections present the recipe for the samples demonstrated in this thesis. Some processes were successful in themselves, and can be used as part of other research directions. Some of the processes failed and should not be used.

Using NbN as Bonding Pads  As we were able to wire-bond directly to the NbN layer, even in 5-nm-thickness, it was tempting to skip step 5 above. However, we found that the 10 – 15 nm thick NbN layer was not conducting through the ferrule-fiber interface, which is full of epoxy glue.
Using FuseLite Ferrules  We tried to using FuseLite ferrules in the building of the optical cavities. These ferrules have 1 cm of fiber glued into them, which can be fused to a longer piece of fiber using a special splicer. The surface of the ferrules turned out to be less smooth than the surface of standard commercially-acquired ferrules, and we did not succeed in fabricating working SNSPDs out of them.

SNSPD Fabrication on Silicon

Tests samples were made on silicon wafer to the end of calibrating the process. The detailed steps of the recipe are as follows:

1. Start with a silicon wafer, covered with a 200-nm-thick SiN layer.

2. Thoroughly clean the wafer in solvents: acetone, methanol, and isopropanol. Clean the wafer for at least 5 minutes per solvent in an ultrasonic bath, and then spin it dry to remove excess isopropanol.

3. Sputter 5 – 15 nm of NbN. This is the most critical step in the process. More details are provided in section 6.3.

4. Pattern contact pads using photolithography. In our case, the mask for this step was ordered from external suppliers. The mask contained 100 × 100 μm² square pads, connected by a 10μm wide line. The pads were connected in groups of four to allow 4-probe contact measurements. In other cases, contact pads should comply with the minimum size required for wire bonding. Photolithography steps:
   a) Dry the sample on a hot plate at 110°C for 10 min.
   b) Deposit 4533 or 1818 photoresist by spinning at 5000 rpm for 60 s.
   c) Bake the sample on 110°C hot plate for 1 min.
   d) Expose the sample to UV light for 5 – 15 s. The exact time is dependent on the microscope.
   e) Develop the photoresist in a TMAH solution (diluted in DI water at a 1:10 ratio) for 40 – 60 s.
   f) Stop the development using DI water for 60 s.
   g) Dry the sample by spinning at 2000 rpm for 1 min.

5. Etch the NbN layer, using the resist as a protective mask. Use a reactive ion etching (RIE) machine, with a mixture of O₂, N₂, and SF₆ gasses. Exact ratios and chamber pressure must be calibrated per machine.

6. Remove the resist using N-methyl-2-pyrrolidon (NMP) for 1 hour. Clean by repeating step 2.
7. Pattern microbridges using an FIB machine. Steps are similar to the one used to pattern meander on fiber, in the previous subsection.

Other Processes
Again, we provide here a few processes with regard to SNSPD on wafer:

Aluminum as Mask for Dry Etching  In some cases we used aluminum as a mask for dry etching, usually after photolithography. The process recipe includes:

1. Obtain a sample consisting of Si covered by a Nb or NbN layer.
2. Sputter 20 nm of aluminum.
3. Create a resist pattern using photolithography, as explained above, or using other resist types.
4. Develop the photoresist in a TMAH solution.
5. Stop the development using DI water.
6. Dry the sample by spinning at 2000 rpm for 1 min.
7. Return the sample to the TMAH solution, and etch the aluminum layer, stop etching using DI water. Use optical microscope to make sure the aluminum layer is fully etched.
8. Remove the resist using solvents (acetone, methanol, isopropanol).
9. Etch the Nb or NbN layer using a dry etching machine: RIE or electron spin resonance (ECR).
10. Remove the aluminum layer using TMAH solution, stop etching using DI water.

The advantage of this method is that aluminum can protect from etching for long times, since it is etched at very slow rate by dry etching. The disadvantage is that patterns from the lithography stage lose precision when being transferred to the aluminum. This procedure cannot be used in conjunction with e-beam lithography as PMMA resists tend to peel off the sample when in contact with TMAH and DI water.

Gold as Mask for Dry Etching  We tried to use gold in a similar way as aluminum in the previous section. Because gold is durable to RIE it is a good mask. However, gold cannot be subsequently removed. The gold etchant we used (see chapter 2.2 also removes NbN, but not uniformly. Therefore, gold can only be used in cases where the mask can remain in place after the RIE process.
CHAPTER 6.  ON-FIBER SINGLE-PHOTON DETECTORS

Wire Bonding

Two types of wire bonding machines were used: gold wire ball bonding, and aluminum wire wedge bonding. In general, ball bonding was less destructive to the sample, and in case of failure, it usually left contact pad unharmed. Unfortunately, we only succeeded to perform gold wire ball bonding on very thick gold pads (layers of 200 nm and more) and PCB covered with a thick copper layer. Aluminum wire bonding was achieved directly on Nb, NbN, and Al pads. Bonding takes a lot of practice and patience, but is very rewarding once the process is well-calibrated to the substrate in use.

EBL of SNSPDs

The common procedure to write SNSPD patterns is to use electron beam lithography (EBL) [21]. In this research we also tried to implement EBL using a Raith E-line machine. We succeeded in fabricating SNSPDs on silicone wafer, but failed to fully reproduce the results on ferrules.

The recipe for SNSPD writing on silicon wafer is as follows:

1. Start with a silicon wafer, covered with a layer of SiN.

2. Thoroughly clean the wafer in solvents: acetone, methanol, and isopropanol. Clean the wafer for at least 5 minutes per solvent in an ultrasonic bath, and then spin dry to remove excess isopropanol.

3. Sputter 5 – 7 nm of NbN. This is the most critical step in the process. More details are given in section 6.3. Note that in this case depositing a thicker NbN will not work, as the resist mask will not be sufficient to protect the metal layer before being completely etched.

4. Deposit the e-beam resist:
   a) Dry the sample on a hot plate at 110°C for 10 min.
   b) Deposit a 495A4 e-beam resist, by spinning at 5000 rpm for 60 s.
   c) Bake the sample on hot plate at 170°C for 5 min.

5. Evaporate thermally 5 nm of chromium to prevent charging. Never use electron beam deposition, as the resist will be exposed.

6. Expose the sample in the e-beam machine.

7. Remove the Cr layer using Cr-7 chromium etchant for 10 s. Excessive use of Cr-7 can harm the e-beam resist. Stop the etching using isopropanol.

8. Develop the resist:
6.A. STEP-BY-STEP FABRICATION RECIPE

a) Develop the resist in an MIBK solution for 40 – 60 s.
b) Stop the development using isopropanol for 20 s.
c) Dry the sample by spinning it at 2000 rpm for 1 min.

9. Etch the NbN layer, using the resist as a protective mask. Use a reactive ion etching (RIE) machine, with a mixture of $O_2$, $N_2$ and $SF_6$ gasses. Exact ratios and chamber pressure must be calibrated per machine. Cooling the RIE stage helps slowing the resist etching time compared to the metallic layer.

10. Remove the resist using N-methyl-2-pyrrolidone (NMP) for 1 hour. Clean by repeating step 2.

An important issue in EBL is the thickness of the resist layer. It is hard to pattern narrow lines in a resist that is too thick. A resist should be at the most two-times thicker than the narrowest lines in the pattern. On the other hand, the metallic layer must be completely etched before the resist is etched, thus imposing a limitation on how thin a resist can be. The resist’s thickness can vary between depositions due to changes in temperature, wafer size, age of the resist, and variance in spinner speed. In order to make sure the resist has the right thickness, we used an Alpha-Step machine; large control patterns were written on the side of the wafer for this purpose, and the resist thickness was measured in every sample before RIE etching.

In order to reproduce writing on ferrules, we fabricated a chuck for the spinner, that can hold one single ferrule in the center. In addition we fabricated a holder to the e-beam stage. Samples covered with resist were baked in an oven in temperature of 170°C for 1 h. The writing results were poor. We could not produce lines narrower than 1 μm.

As we could not measure the thickness of the resist, it was hard to determine if this was the problem. Our ferrules did not fit any of our Alpha-Step or atomic force microscopy machines.
Figure 6.19: An electron beam micrograph of a SNSPD fabricated using EBL. The lines width is 100 nm and the meander’s area is $5 \times 5 \mu m^2$. 
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BIBLIOGRAPHY


התקנים על מוליכים מבוססים ננו חוטים

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לשמניל יחקי של הדירישת לכתבת התואר
דוקטור לפילוסופיה

גייל בכר

הוגש легסנט טכניקו
המחקר עשה בהנחיית פרופסור אייל בוקס
בפקולטה להנדסת חשמל

אני מודה לטכניון – מכון טכנולוגי לישראל, ולמכון ראסל ברי לכנוטכונלוזה על התמיכה הנדיבת בחשדומתי

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תקציר
על-מוליקות היה תופעה של מעבר פאזה, המתרחשת בקופרס והמוסシリーズ. בקירור מסוים חומרים מתפרשים במוליכות. בתנאים התקנים המשלבים קטעים very, very chihuahua, very small. בעבודה זו, אנו בוחנים מספר מהtearDownים של מפלי-مولיקות צרים, דקים וארוכים, המכונים על ידינו "נו-מוליקות". בשל מימדיהם זעירים, עלולים להתקיים "ננו-מוליקות" או "חוטים מהירים". מאידך, במצב המוליך העילי התנגדות שלהם הזעירים,עלולים להתקיים "ננו-מוליקות" או "חוטים מהירים". מאידך, במצב המוליך העילי התנגדות שלהם הזעירים, מגיעה לשיא של הזרמים במעגל, הסטת הזרמים גורמת לפריקת אנרגיה, שתגלית כאות חיצוני גדול. בניגוד למגברי פיצול סטנדרטים, בהתקנים שלנו קיים מנגנון אתחול עצמי, אשר משיב את הגלאי למצב המוליך מיד בסיום התהליך הגילוי. בעבודתנו נסקו שני סוגים של התקנים."

موادHDR לא לינארים מהודי (HDR לא לינארים מהודי), חומרים העשויים גיגה הרץ (-11 תדר רדיול树木ון), המשמשים על ידי פיצול סטנדרטים קיימים, במהלך ביטים קוונטיים. בעבודה מוקדמת במעבדתנו הציגנו תהליך של אי יציבות במהודי ניוביום ניטריד (NbN), המשלב גשרים זעירים. הגשרים הזעירים הם מקטעים צרים ודקים של על-הקרוב לתדר התהודה של המהוד, ומדמו את מקרום ובدمات יוויים. בדיקת המודל שלaits quanto, המודל משמש מספר פעמים בנית מודל המהודי לעיצוב החוזר. גילינו כי קיים עוצמה של הגל המוחזר בתדרי מקרום, וב UIWindowahir ניוביום במחזוריות. בכדי להסביר את תוצאות הניסוי, הצענו מודל פשוט, המייצג את המהוד עיון-יון ממחזוריות.duk, שהNonce המודל הראשון של התזה עוסק בהרחב רק בפ של המודל יוניס rekroph.לב pstmt הסיגנל תופעת האפנון העצminster, ומדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון בין מצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת לגלות תיאודון ביןמצבי ספין שונים. אנו מדגימים את היכולת L Galley

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כלי פוטונים בודדים

כלי פוטונים בודדים, כמו גלאי פוטונים בודדים, וגלאיה אור רגישים, הם gereים במספר רב של תחומי מדע, תקשורת חללית, אסטרונומיה, גילוי תנועה, ריצוף טעינה קוונטית, DNA, בדיקת אטומים מלאכותיים, במספר תחומי מדע אחרים. מדד מרכזי לבחינת איכותו של גלאי פוטונים בודדים הם קצב הגילוי, יעילות הגילוי, הסטייה בתזמון הגילוי (jitter) וקצב הגלוי המזרחי (המכונה קריאות חושך גלאי פוטונים על-נפיקה).

גלאיה עם יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים מקרנים ממקרנים, מצטברים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטומים מלאכותיים, ובראשconciliation יקרות אחר צורות של פוטונים בודדים, הם גלאים ייעודיים לגלואים במגלאים במקרנים ממקרנים, ומורכבים בדיקת יעילות גליוןillow, שתופע את אטום

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