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High intermodulation gain in a micromechanical Duffing resonator

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In this work we use a micromechanical resonator to experimentally study small signal amplification near the onset of Duffing bistability. The device consists of a PdAu beam serving as a micromechanical resonator excited by an adjacent gate electrode. A large pump signal drives the resonator near the onset of bistability, enabling amplification of small signals in a narrow bandwidth. To first order, the amplification is inversely proportional to the frequency difference between the pump and the signal. We estimate the gain to be about 15 dB for our device. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2207490]

Micro/nanoelectromechanical resonators play a key role in microdevices for applications such as sensing, switching, and filtering.[1,2] Understanding nonlinear dynamics in such devices is highly important, both for applications and for basic research.[3–12] The relatively small force needed for driving a microresonator into the nonlinear regime is usually easily accessible, enabling a variety of useful applications such as frequency mixing[13] and frequency synchronization.[14] Since nanoscale displacement detection is highly challenging, it is desirable to implement an on-chip mechanical amplification mechanism. Previously, mechanical amplification has been achieved using parametric amplification.[15,16] Alternatively, amplification could be achieved by using a bifurcating dynamical system.[17,18] In this work, we employ nonlinear frequency mixing near the onset of Duffing bistability to amplify small displacement signals. We demonstrate experimentally high signal gain in this regime and compare with theoretical predictions.

The device under study is a mechanical resonator consisting of a suspended doubly clamped PdAu beam, located adjacent to a static gate electrode. An electron micrograph of the device is shown in the inset of Fig. 1, and its dimensions are given in the figure caption.

The nonlinear dynamics of the fundamental mode of a doubly clamped beam driven by an external force per unit mass $F(t)$ can be described by a Duffing oscillator equation[19] for a single degree of freedom x,

$$\ddot{x} + 2\mu\dot{x} + \omega_0^2(1 + \kappa x^2)x = F(t), \tag{1}$$

where μ is the damping constant, $\omega_0/2\pi$ is the resonance frequency of the fundamental mode, and κ is the cubic nonlinear constant. For small amplitudes, the nonlinearity originates from the axial stress which increases its stiffness ($\kappa > 0$).[20] For higher amplitudes, however, the contribution of the applied electric force, which tends to soften the beam, becomes dominant.

Generally, for resonators driven using a bias voltage applied to a side electrode, Eq. (1) should contain additional parametric terms.[15,21] In our case, however, the prefactors of these parametric terms are at least one order smaller below threshold and thus negligible.

The device has a quality factor $Q=\omega_0/2\mu \approx 2000$ (at 10^{-5} torr) and the fundamental mode resonance frequency $\omega_0/2\pi$ is in the range of 560–630 kHz.

To investigate nonlinear amplification, the resonator is driven by an applied force $F(t)=f_p\cos(\omega_p t)+f_s\cos(\omega_s t+\varphi)$, composed of an intense *pump* with frequency $\omega_p=\omega_0+\sigma$, amplitude f_p, and a small force (called *signal*) with frequency $\omega_s=\omega_p+\delta$, relative phase φ, and amplitude f_s, where $f_s \ll f_p$ and $\sigma, \delta \ll \omega_0$. This is achieved by applying a voltage of the form $V=V_{\mathrm{dc}}+v_p\cos(\omega_p t)+v_s\cos(\omega_s t+\varphi)$, where V_{dc} is a dc bias (employed for tuning the resonance frequency) and $v_s \ll v_p \ll V_{\mathrm{dc}}$. The resonator's displacement has spectral components at ω_p, ω_s, and at the intermodulations $\omega_p \pm k\delta$, where k is an integer. The one at frequency $\omega_i=\omega_p-\delta$ is called the *idler* component, as in nonlinear optics.

In the slowly varying envelope method,[19] the displacement x is written as

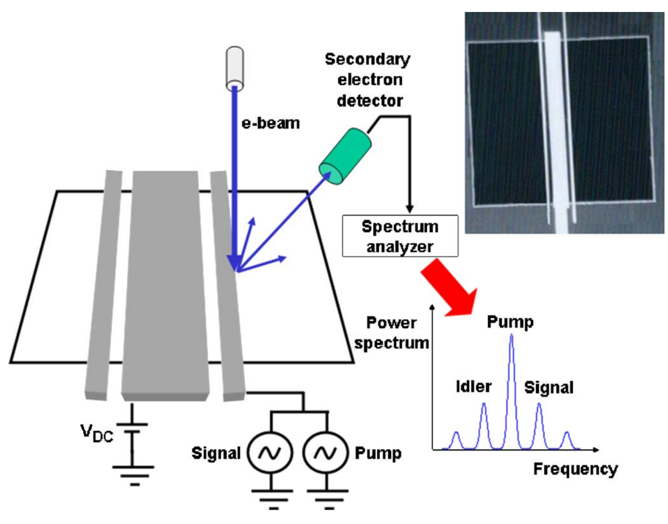

FIG. 1. (Color online) The experimental setup. The inset shows an electron micrograph of the device, consisting of two suspended doubly clamped micromechanical resonators. Each resonator is of length $l=100$ μm, width $w=0.6$ μm, and thickness $t=0.25$ μm, centered around a gate electrode with $d=4$ μm gap. The device is mounted inside a SEM operated in a spot mode to detect the resonator's displacement. The displacement signal is probed by the secondary electron detector and measured using a spectrum analyzer.

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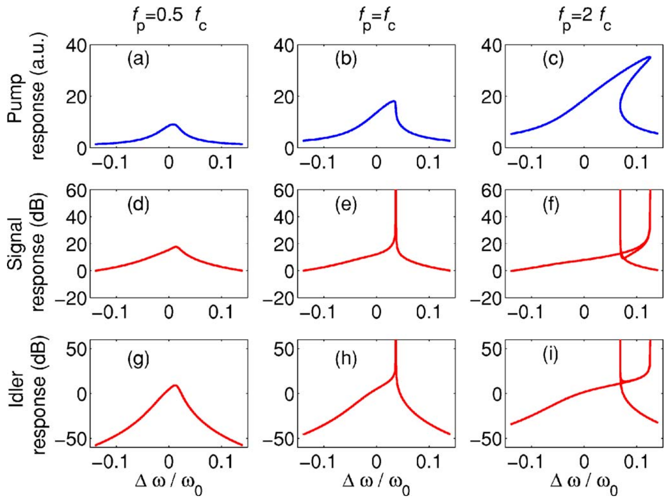

FIG. 2. (Color online) Calculation of the pump, signal, and idler responses ($|a_p|$, $|a_s|$, and $|a_i|$) for vanishing offset frequency δ, shown for subcritical case $f_p=0.5f_c$ [(a), (d), and (g)], critical case $f_p=f_c$ [(b), (e), and (h)], and overcritical case $f_p=2f_c$ [(c), (f), and (i)]. The y axis of the pump is shown in a linear scale while the signal and idler responses are normalized to the signal's excitation amplitude and are shown in a logarithmic scale. The signal and idler responses diverge at the critical point and at the jump points. The parameters for this example are $\kappa=10^{-4}\ m^{-2}$, $\mu=10^2$ Hz, $\omega_0/2\pi=1$ MHz, and $\delta/2\pi=10$ Hz.

$$x(t)=\frac{1}{2}A(t)e^{i\omega_p t}+\text{c.c.}, \qquad (2)$$

where $A(t)$ is a slowly varying function (relative to the time scale $1/\omega_p$). Substituting Eq. (2) in the equation of motion (1) and neglecting the d^2A/dt^2 term yields

$$\frac{dA}{dt}=-\left(\frac{\omega_0}{2Q}+i\sigma\right)A+i\frac{3}{8}\kappa\omega_0A^2A^*+\frac{1}{2i\omega_0}(f_p+f_se^{i(\delta t+\varphi)}). \qquad (3)$$

$A(t)$ can be written as

$$A(t)=a_p+a_se^{i\delta t}+a_ie^{-i\delta t}, \qquad (4)$$

where the complex numbers a_p, a_s, and a_i are the pump, signal, and idler components of $A(t)$, respectively, and $|a_s|$, $|a_i|\ll|a_p|$. Substituting Eq. (4) in Eq. (3) and keeping small terms up to first order leads to

$$a_s=\frac{(1/2\omega_0)f_se^{i\varphi}-(3/8)\kappa\omega_0a_p^2a_i^*}{(3/4)\kappa\omega_0|a_p|^2-\delta-\sigma+i(\omega_0/2Q)}, \qquad (5a)$$

$$a_i=\frac{-(3/8)\kappa\omega_0a_p^2a_s^*}{(3/4)\kappa\omega_0|a_p|^2-\delta-\sigma+i(\omega_0/2Q)}. \qquad (5b)$$

The pump response $|a_p|$ in the absence of any additional signal is shown in Fig. 2, panels (a), (b), and (c).[17] Above some critical driving amplitude f_c, the response becomes a multivalued function of the frequency in some finite frequency range, and the system becomes bistable with jump points in the frequency response. We refer to the onset point of bistability (which is also a saddle-node bifurcation point) as the critical point. When the pump is tuned to the critical point ($\sigma=\sqrt{3}\omega_0/2Q$, $|a_p|^2=8/3\sqrt{3}\kappa Q)$[22] and $\delta\to0$, we expect high amplification of both signal and idler. In this limit

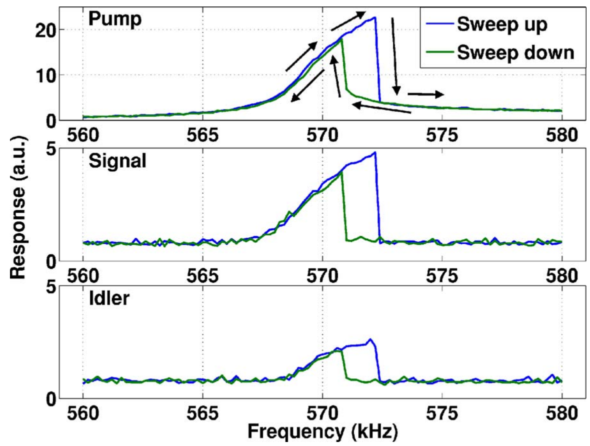

FIG. 3. (Color online) Simultaneous measurement of the pump, signal, and idler spectral components of the mechanical displacement. The excitation frequency is swept upward (blue line) and downward (green line). The arrows in the pump's plot indicate the hysteresis loop. The excitation parameters are pump ac voltage $v_p=0.5$ V, $v_p/v_s=6$, frequency offset $\delta/2\pi=1$ kHz, and $V_{\rm dc}=5$ V. The horizontal axis is the pump frequency for all three plots. The pump, signal, and idler exhibit simultaneous jumps, as expected.

$$|a_s|\simeq|a_i|\simeq\frac{f_s}{2\omega_0\delta}. \qquad (6)$$

Thus, in our model which assumes that $|a_s|$ and $|a_i|$ are small, and takes nonlinearity into account only to lowest order, the amplification diverges in the limit $\delta\to0$. When $|a_s|$ and $|a_i|$ become comparable with $|a_p|$, however, the former assumptions are no longer valid and higher order terms have to be taken into account. The pump, signal, and idler responses were calculated analytically[17] and are shown in Fig. 2. For a small f_p, the signal response is nearly Lorentzian, while for $f_p>f_c$, both signal and idler responses diverge near the jump points.

The resonators are fabricated using bulk nanomachining process together with electron beam lithography.[23] The experimental setup is shown in Fig. 1. Measurement of mechanical vibration is done at room temperature, *in situ* a scanning electron microscope (SEM) where the imaging system of the microscope is employed for displacement detection.[23] The three spectral components ω_p, ω_s, and ω_i of the displacement are measured using a spectrum analyzer.

A typical mechanical response is shown in Fig. 3. The pump frequency is swept upward and then back downward. As expected, we find hysteretic response and simultaneous jumps for the pump, signal, and idler spectral components. In Fig. 4, the mechanical responses of the pump, signal, and idler are depicted as a function of the pump frequency $\omega_p/2\pi$ and the pump ac voltage v_p. For each frequency, the voltage v_p is scanned from 0 to 0.5 V. The results show good agreement with theory. As expected, we observe high signal amplification near the jump points. The amplification can be quantified using a logarithmic scale as

$$G\equiv20\log\left(\left|\frac{a_{s,\text{pump\_on}}}{a_{s,\text{pump\_off}}}\right|\right). \qquad (7)$$

The highest value of G, obtained near one of the jump points, is 15 dB. A comparison with theory is difficult since our model breaks down in the vicinity of the jump points as was explained above. Note, however, that this value is an underestimation of the actual gain due to the nonlinearity of our

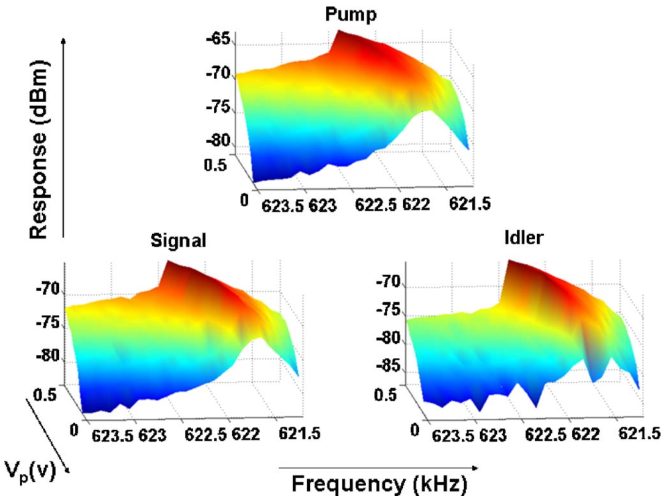

FIG. 4. (Color online) Mesh plots showing the responses of the pump, signal, and idler. The horizontal axis is the pump's frequency ω_p, the diagonal axis is the pump's ac voltage v_p, and the vertical axis is the response (displacement) axis in logarithmic scale. For each frequency, v_p is scanned from 0 to 0.5 V, $v_p/v_s=6$, $\delta/2\pi=100$ Hz, and $V_{dc}=5$ V. Note that the pump response undergoes a jump along a line in the (v_p,ω_p) plane, starting from the bifurcation point. Along the same line, the spectral components of the signal and idler obtain their maximum value.

displacement detection scheme. Since the electron beam diameter is smaller than the displacement amplitude, the output signal is sublinear with respect to displacement.

In conclusion, we have shown that a Duffing micromechanical resonator, driven into the bistability regime, can be employed as a high gain narrow band mechanical amplifier. Strong classical noise squeezing is predicted theoretically for this amplification mechanism when homodyne detection is employed. This will be investigated experimentally in a future work.

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