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Extreme nonlinear phenomena in NbN superconducting stripline resonators

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Abstract

We study giant nonlinear phenomena that emerge near a bifurcation threshold (BT) in a microwave superconducting stripline resonator. Above the BT the system becomes astable and self-sustained modulation of the reflected power off the resonator occurs. The giant nonlinearity manifests itself in an extremely high intermodulation gain, accompanied by a very strong noise squeezing and period doubling of various orders.

Keywords: Nonlinear resonators; Period doubling; Nonequilibrium thermodynamics

The theory of nonlinear dynamics predicts that strong noise amplification, known also as noise rise [1] occurs close to a bifurcation threshold (BT). The amplification is linearly unbounded and only saturates by higher order nonlinear terms [2]. In addition, the same mechanism can be exploited for large amplification of small periodic signals, injected into the system [3]. This mechanism was recently employed in a superconducting (SC) Josephson-junction-based bifurcation-amplifier for state readout of quantum bits [4].

Nonlinear effects in superconductors have, in general, significant implications for both basic science and technology. It may be exploited to explore some important quantum phenomena in the microwave region, such as quantum squeezing [5–7] and experimental observation of the so-called dynamical Casimir effect [8]; Whereas technologically, these effects may allow some intriguing applications such as resonant readout of qubits [9], mixers [10], stochastic resonance [11], and more.

Recently, we have reported on a novel nonlinear phenomenon in SC, in which self-sustained modulation (SM) of a reflected pump tone off a resonator is generated in a SC stripline resonator [12]. The SM only occurs in a certain zone of driven parameters, where the system is astable. We have already shown that near the SM BT, namely near the onset of the instability, the resonator experiences a strong noise amplification [13]. A theoretical model [13] which attributes this phenomenon to a thermal instability in the resonator exhibits a good agreement with the experimental results.

In this Letter we experimentally demonstrate the significance of the SM phenomenon as a generator of several giant nonlinear effects, which manifest themselves at the SM threshold of instability. First, a very strong amplification of periodic signals is demonstrated using intermodulation (IM) measurement. The same measurement technique is used to show period doubling bifurcation (PDB) of various orders. Finally, we demonstrate a phase sensitive deamplification (PSD) which exhibits a strong squeezing factor. Moreover, in the same region, our devices can serve as ultra sensitive and fast detectors of optical radiation [8,14].

Our experiments are performed using a novel device (Fig. 1(B)) which integrates a narrow microbridge into a SC stripline ring resonator. Design considerations, fabrication details as well as resonance modes calculation can be found elsewhere [14]. The microbridge functions as a designed weak-link which has the narrowest dimensions in the resonator. Therefore, the SC current density in the microbridge is much higher than in the rest of the resonator. This current density squared is proportional to the stored-energy inside the resonator. When this energy crosses a certain threshold value, the microbridge switches from the SC phase to the normal-conducting (NC) phase. As a result, the impedance of the microbridge signifi-

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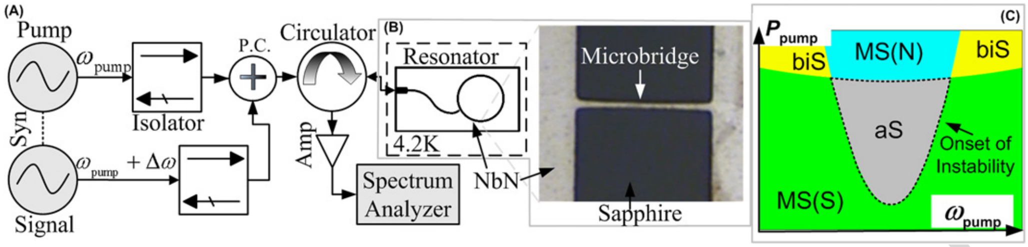

Fig. 1. (Color online.) (A) IM measurement setup. The resonator is stimulated by two phase-locked tones. One, called Pump, is a relatively intense tone which biases the resonator to the astable zone. The other, called Signal, is a relatively weak tone, namely, at list three orders of magnitude weaker than the pump tone. The reflected power off the resonator is measured using a spectrum analyzer (SA). (B) Schematic layout of the resonator. The resonator is made as a stripline ring, having a characteristic impedance of 50 Ω. It is composed of 200 nm thick Niobium Nitride (NbN) deposited on a Sapphire wafer. A weakly coupled feedline is employed for delivering the input and output signals. The first few resonance frequencies fall within the range of 2–8 GHz. The optical image shows the microbridge whose dimensions are $1 \times 10\ \mu m^2$. (C) Schematic diagram showing the stability zones of the resonator as a function of the injected pump power and frequency. The green, pale blue, yellow, and grey colors represent the SC monostable (MS(S)), NC monostable (MS(N)), bistable (biS), and astable (aS) zones, respectively.

cantly increases which, in turn, forces a change in the resonance frequencies and damping rates of the resonator [14]. This mechanism can be used to tune the resonance frequencies of the resonator by applying external perturbations [15], such as infrared illumination [14], on the microbridge. In addition the resonance frequencies can be self-tuned by internal Joule self heating of the microbridge.

Due to the dependence of the stored energy inside the resonator on the resonance frequencies and the damping rates of the resonator, and the dependence of both these parameters on the impedance of the microbridge, the system may have, in general, up to two locally-stable steady-states, corresponding to the SC and NC phases of the microbridge. The stability of each of these phases depends on both the power and frequency parameters of the injected pump tone. In general there exist four different stability zones (Fig. 1(C)) [13]. Two are monostable zones, where either the SC phase or the NC phase is locally stable. Another is a bistable zone, where both phases are locally stable [16,17]. The third is an astable zone, where none of the phases are locally stable. Consequently, when the resonator is biased to this zone, the microbridge is expected to oscillate between the two phases. As the refection coefficient of the resonator differs significantly between these two phases, the oscillations translate into SM of the reflected pump tone. The onset of this instability, namely the BT, is defined as the boundary of the astable zone (see Fig. 1(C)).

The experiments, described in this Letter, are performed while the devices are fully immersed in liquid Helium. The experimental setup, used for the IM experiments, is schematically depict in Fig. 1(A). The SM experiments use an even simpler setup, where the resonator is stimulated with only one monochromatic pump tone at an angular frequency ω_p, and the reflected power off the resonator is measured with a spectrum analyzer.

A typical SM measurement is shown in Fig. 2(A). At low input pump powers, approximately below -33.25 dB m, and at high input powers, approximately above -24.5 dB m, the response of the resonator is linear, namely, the reflected power off the resonator contains a single spectral component at the frequency of the stimulating pump tone. In between the linear regions, there exists a rather large power range, in which regular SM of the reflected power off the resonator occurs. It is realized by rather strong and sharp sidebands, which can extend for several hundred megahertz to both sides of the resonance frequency. The SM frequency, defined as the frequency difference between the pump and the primary sideband, increases as the pump power increases. The lower BT, where the SM starts (using a gradually increasing pump power), spreads on a very narrow power range of approximately 10 nW, where the resonator experiences a strong amplification of the noise floor (noise rise), over a rather large frequency band, and especially around the resonance frequency itself. The upper BT, where the SM ends, spreads over a slightly larger power range than the lower one and has similar, but less extreme characteristics. As expected, the noise rise during the regular SM is negligible, as the transition through the BT is fast [13,18].

IM measurements are one of the common and effective ways to characterize nonlinearities in superconductors [19]. IM, as measured in our devices, is the result of two unequal tones, called Pump and Signal, having close spaced frequencies, being mixed together by a nonlinear system, which produces additional tones at frequencies that are linear combinations (integer multiples) of both. The largest IM products appear at the third order IM mixing tones, known as the Signal and Idler tones, because usually, the second order mixing products do not coincide with any natural resonance frequency of the resonator [20]. The Idler is measured at frequency $f_{\mathrm{Idler}} = 2f_{\mathrm{pump}} - f_{\mathrm{sig}} = f_{\mathrm{pump}} - \Delta f$. Theory predicts that when the injected Pump tone drives the resonator to the BT, both the Signal and the Idler tones undergo equally large amplification [21].

Fig. 2(B) shows a typical IM measurement obtained at the third fundamental resonance mode, namely $f_{\mathrm{pump}} = f_3 = 5.667$ GHz. The Signal tone is deviated by $\Delta f = 800$ Hz from the Pump tone, and thus the Idler and both these tones lie within the resonance band. The pump power biases the resonator to the lower SM BT. In this region, both the Pump, Signal, and Idler tones, as well as higher order mixing products, are easily detected in the reflected power spectrum. The higher order mixing products also spread beyond the scope of this graph and, thus, indicating the strength of the IM nonlinearity at that BT.

The strength of the IM nonlinearity can be qualitatively characterized by defining two parameters; IM Signal and Idler gains, defined as the ratio of the output Signal and the output Idler powers to the input Signal power, respectively. Fig. 2(D)

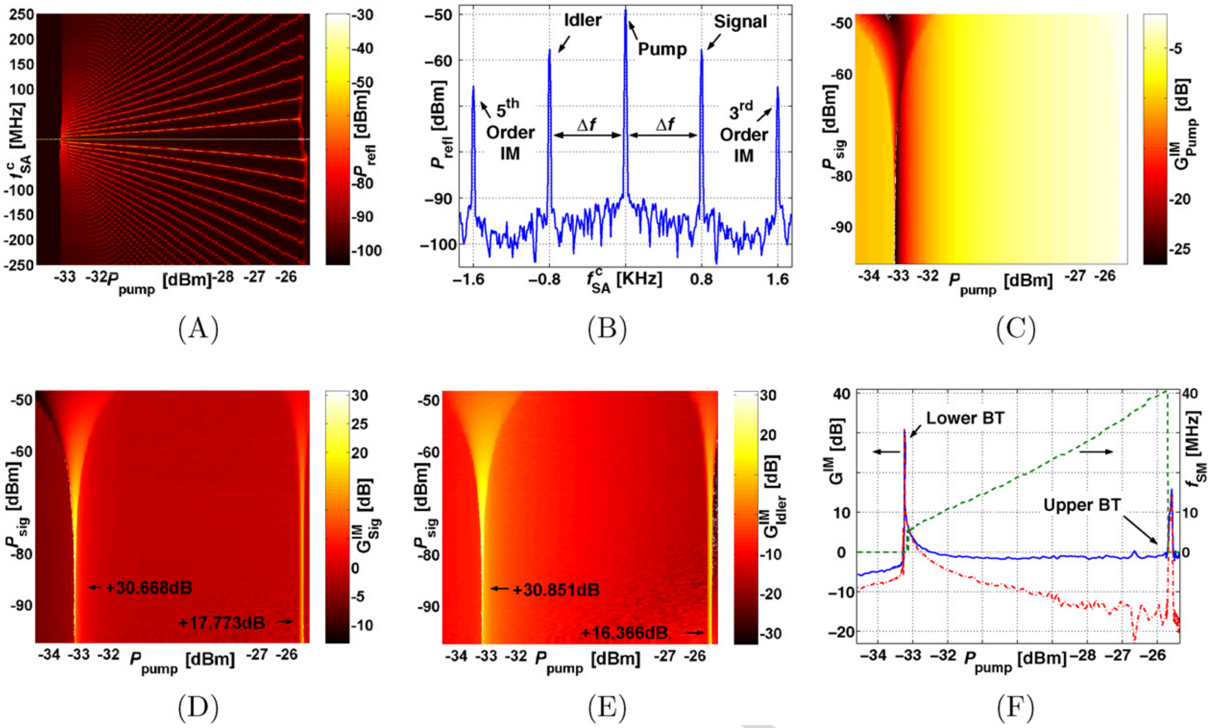

Fig. 2. (Color online.) Typical SM (A) and IM measurements (B)–(E). (A) The reflected power P_{refl} is plotted as a function of the input Pump power and the measured frequency f_{SA}, centralized on the third fundamental resonance frequency f_3 ($f_{\mathrm{SA}}^c = f_{\mathrm{SA}} - f_3$), while the resonator is stimulated with a single monochromatic tone at f_3. (B) The reflected power is plotted as a function of the measured frequency centralized on f_3. (C), (D), (E) Pump, Signal and Idler IM gains, respectively, measured as a function of the input Pump and input Signal powers. (F) Signal (solid-blue) and Idler (dashed-dotted red) IM gains versus the SM frequency (dotted-green) as a function of input Pump power.

and (E) plot these gains, respectively, as a function of both the input Pump and input Signal powers. Two areas of strong amplification, indicated by bright shaded colors, are easily noticed. The pump powers at which they occur are referred to as the lower and upper power-thresholds. The maximum amplifications achieved by the Signal tone at the lower and upper power-thresholds are 30.66 dB and 17.77 dB, respectively; whereas the maximum amplifications achieved by the Idler tone are 30.85 dB and 16.36 dB, respectively. The Signal and Idler gains are approximately equal, as expected. To emphasize the strength of these amplifications we note that, usually, no amplification greater than unity (0 dB) is achieved in IM measurements with SC resonators [20,21]. Fig. 2(C) shows the corresponding gain of the Pump tone. It experiences a strong absorption (pump depletion) at the lower power threshold, where the large amplification of the Pump and Signal tones takes place. It also experiences an increased absorption at the upper power threshold, but it is visually small on this scale and, therefore, harder to be noticed.

The correlation between simultaneously measured SM and IM phenomena is demonstrated in Fig. 2(F). The solid-blue (dashed-dotted red) curve shows the IM Signal (Idler) gain as a function of the input pump power, while the input signal power equals −86 dB m (−79.6 dB m), for which the maximum of the gain is measured. The dashed-green curve shows the SM frequency as a function of the input pump power. The comparison clearly shows that the strong IM gain is achieved at the same BT powers as the SM phenomenon. This finding indicates that the correlation between these two phenomena is strong, and proves that the observed large IM gain does not originate from various other possible nonlinear mechanisms, common in superconductors [17]. The narrow power range, at which the Signal and Idler gains are achieved, which is by most 5 nW wide, emphasizes the sharpness of the BT in our device.

It has been found that, in some nonlinear systems, the transition to chaos can occur via consecutive PDB instabilities of various orders [22–25]. It was shown that near the onset of PDB, any dynamical system can be used to amplify perturbations near half the fundamental frequency [26]. PDB can be indirectly observed in some of our devices, while performing IM measurements.

Fig. 3 has four subplots, each showing an IM measurement, in which PDB is observed. The strong Pump and weak Signal powers are set to the lower BT power, and the reflected Pump, Signal, Idler, and higher order mixing products, are easily observed (labeled at subplot (iv)). In addition, looking at subplot (i), we observe a new type of reflected tones which are found at half-integer multiples of the Pump and Signal frequencies. For example, the two labeled tones are found at $f_{S1}^{i} = (3f_{\mathrm{pump}} - f_{\mathrm{sig}})/2$ and $f_{S2}^{i} = (f_{\mathrm{pump}} + f_{\mathrm{sig}})/2$. This measurement provides clear evidence that a period doubling of the second order occurs in the resonator. Subplots (ii) and (iii) show additional measurements, in which period doubling of the forth and third orders occur, respectively. In addition, subplot (iv) shows a measurement in which a chaotic-like behavior is observed. The behavior is characterized by a strong and broadband amplification of the noise floor and a high absorption of the pump power. The above described measurements were taken while setting the Pump power to the threshold power and sweeping the weak Signal power. Additional measurements showed that the system can self-switch between the various period doubling states even when all deliberately applied external excitations are constant. This occurs due to the presence of a quasi-periodic very low frequency noise in the resonator. At this point the origin of this noise is yet unclear.

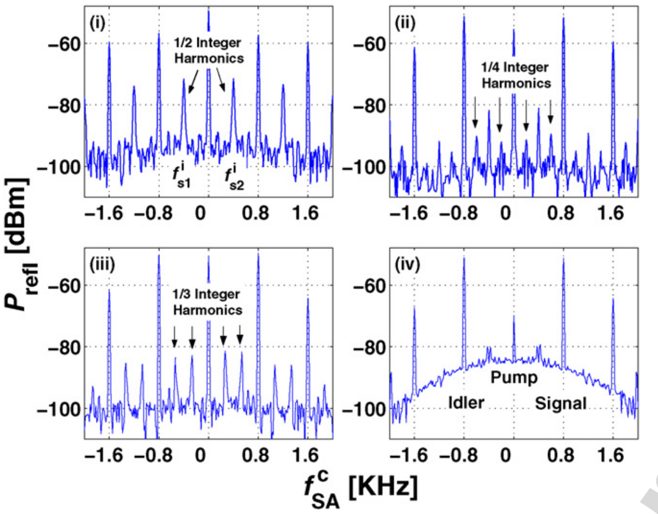

Fig. 3. (Color online.) Period doubling captured during IM measurement. The reflected power is plotted as a function of the measured frequency f_{SA} centralized on f_3 ($f^c_{SA} = f_{SA} - f_3$). It shows the Pump, Signal, and Idler tones. In addition, it shows the mixing products of (i) half, (ii) quarter, and (iii) one third subharmonics of the Pump and Signal tones. Panel (iv) shows chaotic-like behavior characterized by a strong and broadband amplification of the noise floor.

In general, a strong IM gain can establish correlations between the output tones at $f_{pump} \pm \Delta f$, where $\Delta f = |f_{sig} - f_{pump}|$ [27]. These correlations are measured by employing a homodyne detection method. Namely, the output of the resonator is delivered to an external mixer and mixed with a local oscillator (LO), whose phase is locked to the phase of the pump tone. Due to the correlation, the measured spectral power becomes periodically dependent on the relative phase ϕ_{LO} between the LO and the pump [6] and, as a result, can have a reduced power relative to a similar measurement where no such correlations exist. This phenomenon is called PSD [28] or squeezing, for thermal noise [29], weak signals [30], or quantum fluctuations [5] reduction. The theory is detailed in [27], and summarized in [31]. In our resonators we observe PSD of both the fluctuation noise and the spectral power of the sidebands generated by the SM phenomenon.

A typical PSD measurement, as obtained using the setup described in Fig. 4(A), is shown in Fig. 4(B). The solid-blue curve shows the strongest reflected power spectrum. Due to the mixing process, the pump tone is down-converted to dc and the SM sidebands are down-converted relatively to the pump. We refer to this measurement as taken with a zero LO phase, $\phi_{LO} = 0$. The dashed-red curve is taken with $\phi_{LO} = \pi/2$. The deamplification of the reflected power spectrum, relatively to $\phi_{LO} = 0$ curve is clear. Panel (C) shows a similar measurement, taken for continuous ϕ_{LO} values. The dependence of the reflected power spectrum on ϕ_{LO} is clearly observed, where the phase period equals π, as expected. We define the squeezing factor as the ratio between a measured spectral component at its maximum value and at its minimum value. Panel (D) shows a colormap of this squeezing factor as a function of the measured frequency and the pump power. Strong and broadband squeezing occurs at the two BT powers, where the strong IM gain is measured. In

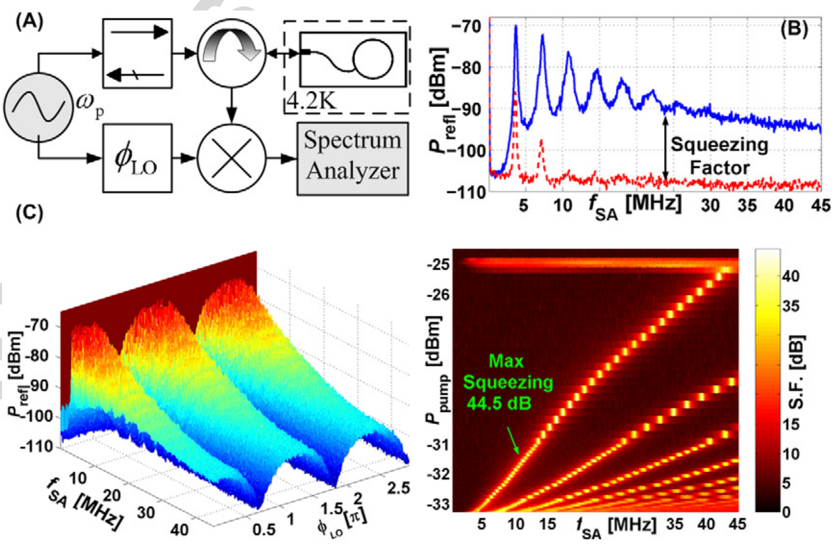

Fig. 4. (Color online.) (A) PSD measurement setup, which utilizes homodyne detection measurement scheme. The resonator is stimulated by a single monochromatic pump and the reflected power is fed to an external mixer, driven by a local oscillator (LO), having the same frequency as the pump and an adjustable phase. The output of the mixer is measured by a SA in a frequency band of 45 MHz starting from DC. (B) A typical PSD measurement. Reflected power as a function of the measured frequency for $\phi_{LO} = 0$ (solid-blue) and $\phi_{LO} = \pi/2$ (dashed-red). The same measurement for a continuous LO phase is shown in (C). (D) Squeezing factor as a function of the measured frequency and the input pump power.

addition, the SM sidebands undergo an even stronger squeezing, throughout the astable zone, where a maximum value of 44.5 dB has been measured.

In conclusion, our devices exhibit an extreme nonlinear behavior which manifests itself in various effects, in correlation with the SM phenomenon. Strong IM gain of both the Signal and Idler tones is measured in all of our devices at the same BT where the SM emerges. Correlated PDB of various orders as well as chaotic-like behavior occur at some of our devices. In addition, PSD is measured, and shows a strong squeezing factor in correlation with the IM strong gain. The outcome of this three correlated phenomena is a very strong, phase sensitive amplifier. Such a device is highly suitable for quantum data-processing systems and for quantum state readout schemes.

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References

[1] K. Wiesenfeld, J. Stat. Phys. 38 (1985) 1071.
[2] Y. Kravtsov, E. Surovyatkina, Phys. Lett. A 319 (2003) 348.
[3] K. Wiesenfeld, B. McNamara, Phys. Rev. A 33 (1) (1986) 629.
[4] I. Siddiqi, R. Vijay, F. Pierre, C.M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, M.H. Devoret, Phys. Rev. Lett. 93 (2004) 207002.
[5] R. Movshovich, B. Yurke, P.G. Kaminsky, A.D. Smith, A.H. Silver, R.W. Simon, M.V. Schneider, Phys. Rev. Lett. 65 (12) (1990) 1419.
[6] B. Yurke, E. Buks, J. Lightwave Tech. 24 (2006) 5054.
[7] E. Buks, B. Yurke, Phys. Rev. A 73 (2005) 023815.
[8] E. Arbel-Segev, B. Abdo, O. Shtempluck, E. Buks, quant-ph/0606099.
[9] J.C. Lee, W.D. Oliver, T.P. Orlando, K.K. Berggren, IEEE Trans. Appl. Superconduct. 15 (2005) 841.
[10] D.W. Floet, E. Miedema, T.M. Klapwijk, J.R. Gao, Appl. Phys. Lett. 74 (1999) 433.
[11] B. Abdo, E. Arbel-Segev, O. Shtempluck, E. Buks, cond-mat/0606555.
[12] E. Arbel-Segev, B. Abdo, O. Shtempluck, E. Buks, cond-mat/0607259.
[13] E. Segev, B. Abdo, O. Shtempluck, E. Buks, J. Phys.: Condens. Matter 19 (2007) 096206, cond-mat/0607261.
[14] E. Arbel-Segev, B. Abdo, O. Shtempluck, E. Buks, IEEE Trans. Appl. Superconduct. 16 (3) (2006) 1943.
[15] D. Saeedkia, A.H. Majedi, S. Safavi-Naeini, R.R. Mansour, IEEE Microwave Wireless Compon. Lett. 15 (8) (2005) 510.
[16] B. Abdo, E. Segev, O. Shtempluck, E. Buks, IEEE Trans. Appl. Superconduct. 16 (2006) 1976.
[17] B. Abdo, E. Segev, O. Shtempluck, E. Buks, Phys. Rev. B 73 (2006) 134513.
[18] Y.A. Kravtsov, S.G. Bilchinskaya, O.Y. Butkovskii, I.A. Rychka, E.D. Surovyatkina, JETP 93 (2001) 1323, Zh. Eksp. Teor. Fiz. 120 (2001) 1527.
[19] B. Abdo, E. Segev, O. Shtempluck, E. Buks, Appl. Phys. Lett. 88 (1) (2006) 022508.
[20] C.C. Chin, D.E. Oates, G. Dresselhaus, M.S. Dresselhaus, Phys. Rev. B 45 (9) (1992) 4788.
[21] R. Monaco, A. Andreone, F. Palomba, J. Appl. Phys. 88 (5) (2000) 2898.
[22] R.M. May, Nature 261 (1976) 459.
[23] M.J. Feigenbaum, J. Stat. Phys. 19 (1) (1978) 25.
[24] P.S. Linsay, Phys. Rev. Lett. 47 (19) (1981) 1349.
[25] I. Balberg, H. Arbell, Phys. Rev. E 49 (1) (1994) 110.
[26] K. Wiesenfeld, B. McNamara, Phys. Rev. Lett. 55 (1) (1985) 13.
[27] B. Yurke, Phys. Rev. A 32 (1) (1985) 300.
[28] M.F. Bocko, J. Battiato, Phys. Rev. Lett. 60 (1988) 1763.
[29] B. Yurke, P.G. Kaminsky, R.E. Miller, E.A. Whittaker, A.D. Smith, A.H. Silver, R.W. Simon, Phys. Rev. Lett. 60 (9) (1988) 764.
[30] R. Almog, S. Zaitsev, O. Shtempluck, E. Buks, cond-mat/0607055, Phys. Rev. Lett., in press.
[31] B. Yurke, P.G. Kaminsky, R.E. Miller, E.A. Whittaker, A.D. Smith, A.H. Silver, R.W. Simon, IEEE Trans. Magn. 25 (9) (1989) 1371.