

# Unusual Nonlinear Dynamics Observed in NbN Superconducting Microwave Resonators

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**Abstract.** We report on some unusual nonlinear dynamics observed in several NbN superconducting stripline microwave resonators. Among the nonlinear effects observed: abrupt bifurcations within the resonance lineshape, hysteretic behavior, hysteresis loops changing direction, resonance shift and critical coupling. Weak links forming at the boundaries of the columnar structure of the NbN films are hypothesized as the main origin of the nonlinearities.

## 1. Introduction

Nonlinear effects in superconductors have attracted an extensive research effort along the years [1]. A large part of the interest aims at studying the variety of nonlinear origins in superconductors, and considering ways to minimize their negative impact, as these effects, especially in high temperature superconductors, are known to degrade considerably the performance of planar superconducting microwave devices designed for telecommunication area [2].

The nonlinear dynamics observed in our NbN superconducting resonators are qualitatively different from the commonly reported nonlinear effects, namely the Duffing oscillator nonlinearity [3]. Investigating these nonlinear effects and their origin may be useful for the study of nonlinear mechanisms in superconductors in general, and for the experimental demonstration of some interesting quantum phenomena in the microwave regime in particular [4].

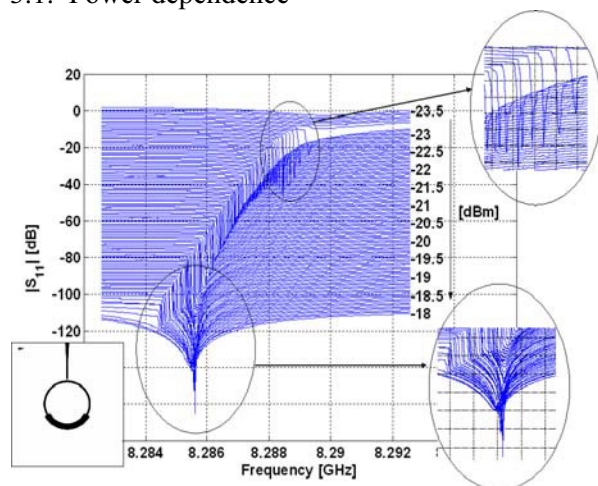
## 2. Design and Fabrication

In this paper we will present the results of two nonlinear NbN resonators. The resonators were dc-magnetron sputtered on 34mmX 30mmX 1mm sapphire substrates near room temperature, and were assembled using standard stripline geometry. Layouts of the center strip of the two resonators, which we will refer to them by the names B1 and B2 for simplicity, are depicted in the insets of Figures 1 and 2 respectively. The gap between the resonator and its feedline was set to 0.4 mm in B1 resonator and 0.5 mm in B2. General design considerations and sputtering details are presented elsewhere [5].  $T_c$  measured for B1 and B2 are 10.7K and 6.8K respectively, whereas  $\rho$  measured for B2 resonator is 384 $\mu\Omega$ cm. These relatively low  $T_c$  and high normal resistivity can be partially attributed to the relatively high concentration of Nitrogen applied in the Ar/N<sub>2</sub> mixture during the sputtering process [6].

### 3. Measurements

The measurements presented here were obtained by measuring the reflection parameter ( $S_{11}$ ) of a vector network analyzer. All measurements except the one presented in subsection (3.2) were conducted at 4.2K.

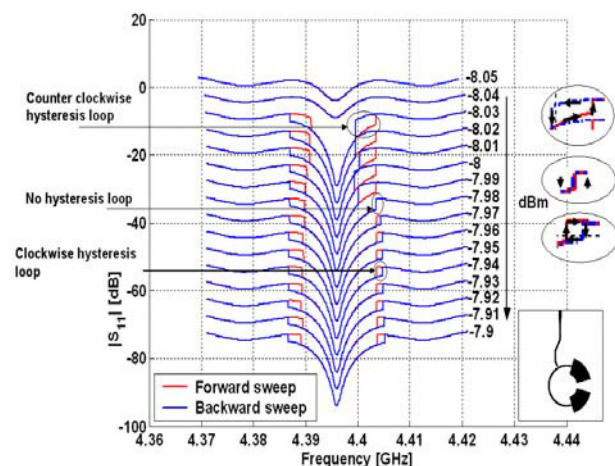
#### 3.1. Power dependence



**Figure 1.**  $S_{11}$  measurement of B1 resonance  $\sim 8.288$ GHz at low input powers within 10MHz span. The measurement exhibits extraordinary nonlinear effects. The resonance curves corresponding to different input powers were shifted vertically by a constant offset for clarity.

Figure 1 shows some of the unusual nonlinear dynamics observed in these resonators as a function of input power. The resonance measured is the third mode of B1 resonator  $\sim 8.288$  GHz. The bifurcations appear in the resonance curve at relatively low input power threshold  $\sim 23.25$  dBm, 2-3 orders of magnitude lower than Nb for example [5]. The bifurcations also occur abruptly and hence are very sensitive to input power unlike Duffing oscillator nonlinearity which builds up gradually [3]. Moreover as the input power is increased the bifurcation frequency is red shifted and the bifurcation magnitude changes, whereas at a higher input power, critical coupling condition is measured, which is a coupling state where at resonance the reflected power becomes practically zero at the resonator port. Increasing the input power further causes the bifurcation to disappear and the resonance lineshape to become more symmetrical.

#### 3.2. Frequency hysteresis

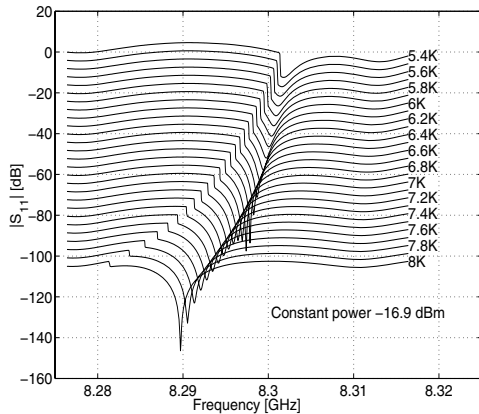


**Figure 2.** Forward and backward frequency sweep measurement, performed using network analyzer, measuring B2 second mode nonlinear resonance. The red line represents a forward sweep whereas the blue line represents a backward sweep. The graphs exhibit clear hysteresis loops forming at the vicinity of the bifurcations, and hysteresis loop changing direction as the input power is increased. The resonance curves corresponding to different input powers were shifted vertically by a constant offset for clarity.

Sweeping the frequency in the forward and backward directions reveals a hysteretic behaviour of the resonance lineshape in the vicinity of the bifurcations. In figure 2 we show the result of such a

measurement performed on B2 second resonance frequency  $\sim 4.39$  GHz. Increasing the input power exhibits yet another interesting phenomenon. At input power of  $-7.98$  dBm the hysteresis loop at the right side bifurcation changes its circulation direction from counter clockwise to clockwise. This feature was also encountered in B1 resonances and at the left side bifurcation of this resonance as well, though at a different power level. Such a unique hysteretic behavior can not be explained fully in terms of one-dimensional Duffing oscillator nonlinearity [3].

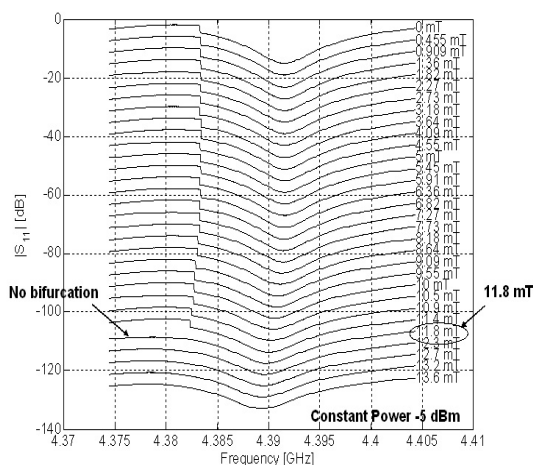
### 3.3. Temperature dependence



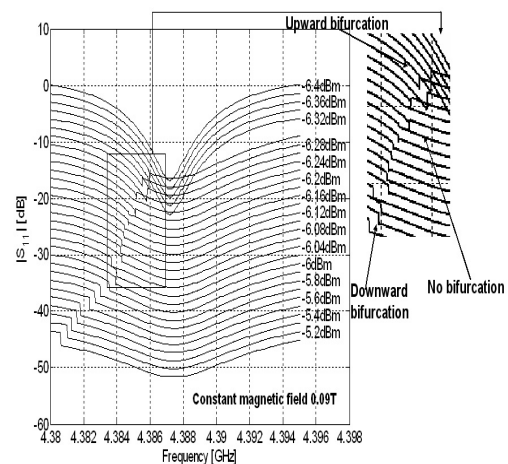
**Figure 3.** The plot exhibits the nonlinear resonance frequency response of B1 third mode measured under constant input power of  $-16.9$  dBm, while increasing the temperature from  $5.4\text{K}$  through  $8\text{K}$  in steps of  $0.1\text{K}$ .

In order to examine the dependence of B1 third resonance frequency on ambient temperature, we have set the input power to a constant power level of  $-16.9$  dBm, and varied the temperature between  $5.4\text{K}$  through  $8\text{K}$  in steps of  $0.1\text{K}$ . The measurement result is shown in figure 3, where the effect of the temperature varying on the bifurcation magnitude and frequency is evident. Moreover as the ambient temperature is increased the minima frequency is red shifted and the resonance curve becomes more symmetrical. The observed resonance shift can be generally explained in terms of increase in the penetration depth of the film as the temperature is increased.

### 3.4. Magnetic field dependence



**Figure 4.** Increasing the magnetic field gradually from zero while applying a constant input power level of  $-5$  dBm. The measured curves were shifted vertically by a constant offset for clarity.



**Figure 5.** B2 nonlinear resonance response measured under a constant magnetic field of  $0.09\text{T}$ , while increasing the input power. The measured curves were shifted vertically by a constant offset for clarity.

Other interesting nonlinear features were obtained while applying perpendicular magnetic field. In figure 4 the input power was set to a constant value of -5 dBm, whereas the magnetic field was increased from zero in small steps. As the magnetic field reaches about 11.8mT the bifurcation at the left side of the resonance curve vanishes. In contrast, in figure 5 the magnetic field was held constant (0.09T) whereas the input power was varied in small steps. By inspecting the figure one can observe a power range at which there are no bifurcations, followed by two sequential power ranges at which the corresponding resonance curves contain a bifurcation heading upward and downward respectively. These two measurements indicate that the nonlinear mechanism acting on these resonators is highly sensitive to magnetic fields.

#### 4. Weak links in the NbN columnar structure

Weak links forming at the boundaries of the NbN columnar structure (verified using scanning electron microscope micrographs [5]) are hypothesized as the main physical mechanism responsible for the observed effects. The hypothesized weak links may be either unintentionally grown Josephson junctions at the NbN columnar boundaries, which may explain the observed input power and magnetic field sensitivity [7], or weak superconducting points switching to normal state via a very fast local heating mechanism similar to hot spots [8]. Further discussion and experimental data supporting either mechanism are brought elsewhere [5].

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#### References

- [1] Halbritter J 1990 *J. Appl. Phys.* **68** 6315  
Golosovsky M A, Snortland H J and Beasley M R 1995 *Phys. Rev. B.* **51** 6462  
Oates D E, Hein M A, Hirst P G, Humphreys R G, Koren G and Polturak E 2002 *Physica C* **462** 372
- [2] Dahm T and Scalapino D J 1997 *J. Appl. Phys.* **81** 2002
- [3] Chen C C, Oates D E, Dresselhaus G and Dresselhaus M S 1992 *Phys. Rev. B.* **45** 4788  
Oates J H, Shin R T, Oates D E, Tsuk M J and Nguyen P P 1993 *IEEE Trans. on Appl. Superconduct.* **3** 17
- [4] Yurke B and Buks E 2005 *Preprint* quant-ph/0505018  
Abdo B, Segev E, Shtempluck O and Buks E 2005 *Preprint* cond-mat/0507056  
Abdo B, Segev E, Shtempluck O and Buks E 2005 *Preprint* cond-mat/0501236
- [5] Abdo B, Segev E, Shtempluck O and Buks E 2005 *Preprint* cond-mat/0501114  
Abdo B, Segev E, Shtempluck O and Buks E 2005 *Preprint* cond-mat/0504582
- [6] Bacon D D, English A T, Nakahara S, Peters F G, Schreiber H, Sinclair W R and Dover R B 1983 *J. Appl. Phys.* **54** 6509
- [7] Prance H, Clark T D, Whiteman R, Prance R J, Everitt M, Stiffel P and Ralph J F 2004 *Preprint* cond-mat/0411139  
Whiteman R, Diggins J, Schollmann V, Clark T D, Prance R J, Prance H and Ralph J F 1997 *Phys. Lett. A* **234** 205
- [8] Skocpl W J, Beasley and Tinkham 1974 *J. Appl. Phys.* **45** 4054  
Gurevich A VI and Mints R G 1987 *Rev. Mod. Phys.* **59** 941