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# Prospects of employing superconducting stripline resonators for studying the dynamical Casimir effect experimentally

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**Abstract**

We discuss the prospects of employing an NbN superconducting microwave stripline resonator for studying the dynamical Casimir effect experimentally. Preliminary experiments, in which optical illumination is employed for modulating the resonance frequencies of the resonator, show that such a system is highly promising for this purpose. In addition, we discuss the undesirable effect of heating which results from the optical illumination, and show that degradation in noise properties can be minimized by employing an appropriate design.


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The term dynamical Casimir effect (DCE) usually refers to the problem of changing the quantum state of an electromagnetic (EM) field in a cavity with periodically moving walls. The quantum theory of electrodynamics predicts that, under appropriate conditions, photons should be created in such a cavity out of the vacuum fluctuations [1,2]. Such motion-induced radiation is closely related to the Unruh–Davies effect, which predicts that an observer of the EM field in a uniformly accelerating frame would measure thermal radiation with an effective temperature given by $\hbar a/2\pi k_B c$, where $a$ is the acceleration, $k_B$ is the Boltzmann's constant, and $c$ is the light velocity in vacuum. Moreover, the equivalence principle of general relativity relates the later effect with the so-called Hawking radiation of black holes [3–7].

So far, the DCE has not been verified experimentally [8,9]. It turns out that creation of photons in the case of a cavity with moving walls requires that the peak velocity of the moving walls must be made comparable to light velocity; a task which is experimentally extremely difficult [10]. Most efficient photon creation is achieved by employing primary parametric resonance, where the cavity walls performs harmonic oscillates at twice the resonance frequency of one of the cavity modes [11,12]. The angular resonance frequency $\omega_r$, in this case, varies in time according to $\omega_r(t) = \omega_0[1 + \xi \cos(2\omega_0 t)]$. The system's response to such an excitation depends on the dimensionless parameter $\xi Q$, where $Q$ is the quality factor of the resonator [13]. When $\xi Q < 1$, the system is said to be in the sub-threshold region, where the thermal average number of photons is time independent. On the other hand, when the system is above threshold, $\xi Q > 1$, the average number of photons should grow exponentially. Achieving the condition $\xi Q > 1$ requires that the shift in the resonance frequency exceeds the width of the resonance mode.

An alternative method for realizing the DCE was pointed out by Yablonovitch [14], who proposed that modulating the dielectric properties of a material in an EM cavity might be equivalent to moving its walls. As a particular example, he considered the case of modulating the dielectric constant $\epsilon$ of a semiconductor by optical pulses that create electron–hole pairs. The modulation frequency can be made sufficiently high as it is only limited by the recombination time of electron–hole pairs, which can be relatively fast in some semiconductors [15].

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Based on this idea, a novel experimental approach for the detection of the DCE was recently proposed [8,16]. This approach, however, has a major drawback [2]. The change in the dielectric constant $\Delta\epsilon = \Delta\epsilon' + i\Delta\epsilon''$ that occurs due to creation of electron–hole pairs in a semiconductor, can be found by employing the Drude model [17]. In the microwave region one finds that $\Delta\epsilon'/\Delta\epsilon'' \simeq \omega\tau$ (see Eq. (1.35) of Ref. [17]), where $\tau$ is the momentum relaxation time, and $\omega$ is the angular frequency. Unfortunately, $\omega\tau \ll 1$ for all known semiconductors in the microwave region, and consequently, unless the resonator is carefully designed [2], exciting charge carriers mainly leads to undesirable broadening of the resonance peaks while the frequency shift is expected to be relatively small.

We propose an alternative approach, in which the modulation of $\epsilon$ is implemented with a superconductor rather than a semiconductor. Optical radiation in this case is used to modulate the relative density of superconducting electrons and that of normal electrons. The resultant change in the dielectric constant $\Delta\epsilon$ can be found from the two-fluid model [18], according to which $\Delta\epsilon$ depends on the ratio between the London length $\lambda$ and the skin depth of normal electrons $\delta$. This ratio, according to London's theory, is given by $\lambda_0/\delta = (\omega\tau/2)^{1/2}$, where $\lambda_0$ is the London length at zero temperature (see Eqs. (14.21) and (34.9) of Ref. [17]). Consequently, one finds that $\Delta\epsilon'/\Delta\epsilon'' \simeq 1/\omega\tau$ [19], thus for most superconductors $\Delta\epsilon'/\Delta\epsilon'' \gg 1$ in the microwave region. This property of superconductors significantly facilitates achieving parametric gain and demonstrating the DCE by optically modulating $\epsilon$ since the frequency shift can be made much larger in comparison with an undesirable peak broadening (see Eq. (A4) in Ref. [20]). Indeed, quasi-static resonance frequency shift by optical radiation [21,22], or high-energy particles [23] (for which the required condition $\xi Q \cong 1$ has been achieved) has already been demonstrated.

In the present Letter we discuss the prospects of employing a superconducting stripline resonator for experimentally studying the DCE. Our preliminary results show that such a system is highly promising for this purpose. In particular we show that achieving parametric gain by optically modulating the resonance frequency is feasible. The quantum nature of the DCE requires operating at vary low temperatures. On the other hand, the optical modulation might induce undesired heating of the illuminated section. Therefore, in the last part of this Letter, we theoretically analyze the expected effect of the induced heating on the noise properties of the system. We simplify the analysis by focusing on the subcritical region $\xi Q < 1$ and study the conditions for achieving noise squeezing [15,24–26]. Our results show that the undesirable effect of heating can be minimized by employing an appropriate design, in which the resonator is strongly overcoupled.

We employ a novel configuration, in which a hot electron detector (HED) is implemented as an integrated part of a microwave superconducting NbN stripline resonator. The absorption of an optical pulse in the HED generates hotspots, which are basically islands of normal-conducting domains, with a temperature above the critical one, coexisting with the surrounding superconductivity [27]. As a result, the impedance of the HED is substantially modified [28]. Both the change in the resistive and inductive parts of the HED impedance contributes to a resonance shift $\Delta f$ [23,29], whereas the undesirable change in the damping rate is relatively small. The modulation frequency is limited by the relaxation process of high-energy quasi-particles, also called 'hot-electrons', giving their energy to the lattice, and recombining to form Cooper pairs. Recent experiments with such photodetectors have demonstrated an intrinsic thermal relaxation time on the order of tens of picoseconds (see [30] and references therein), thus the modulation frequency can be made sufficiently high.

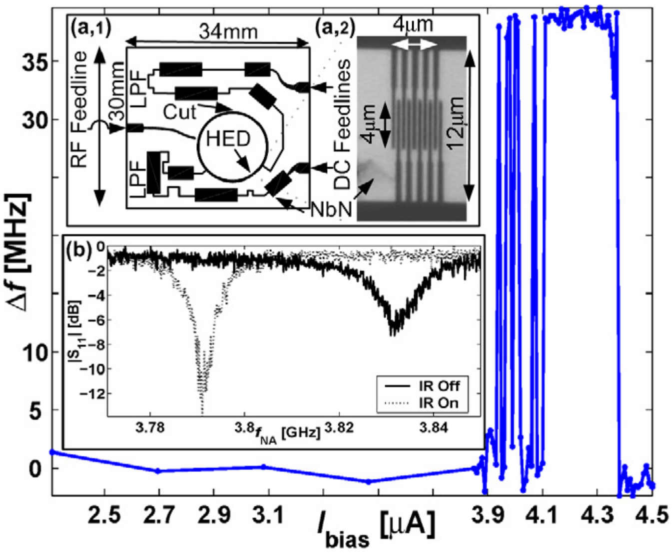


Fig. 1. Resonance frequency shift $\Delta f$, induced by IR laser illumination, plotted at various dc current bias values. Inset (a) shows the layout of the device and an optical image of the HED. Inset (b) shows the reflection coefficient ($|S_{11}|$) measurement in the vicinity of the second resonance mode with (dotted) and without (solid) IR laser illuminating the HED, while a sub-critical dc current of 4.14 μA is applied through the HED.

The circuit layout is illustrated in inset (a, 1) of Fig. 1. The resonator is designed as a stripline ring having a characteristic impedance of 50 Ω. It is composed of 8-nm-think NbN film deposited on a Sapphire wafer. The first few resonance frequencies fall within the range of 2–8 GHz. A feedline, weakly coupled to the resonator, is employed for delivering the input and output signals. A HED is integrated into the structure of the ring. Its angular location, relative to the feedline coupling location, maximizes the RF current amplitude flowing through it in one of the resonance modes and thus maximizes its coupling to that mode. The HED has a meander shape (inset (a, 2) of Fig. 1) that consists of nine strips. Each strip has a characteristic area of $0.15 \times 4\ \mu m^2$ and the strips are separated one from another by approximately 0.25 μm [31]. The HED operating point can be maintained by applying dc bias. The dc bias lines are designed as two superconducting on-chip low-pass filters (LPF). A cut of 20 μm is made in the perimeter of the resonator, to force the dc bias current flowing through the HED. A fiber optic cable is used to guide laser illumination to the HED. Further design considerations, fabrication details as well as calculation of normal modes can be found elsewhere [29]. Measurements are carried out with a fully immersed sample in liquid Helium.

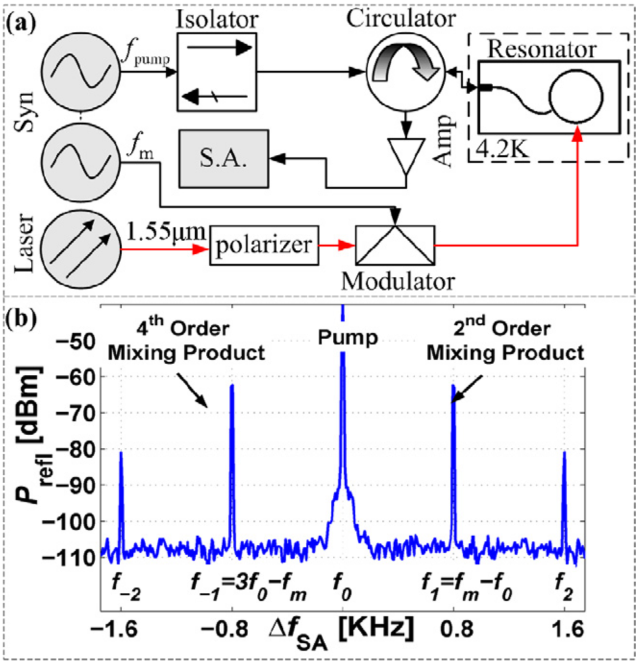


Fig. 2. (Color online.) (a) Setup used for parametric excitation measurement. (b) The reflected power versus the measured frequency $f_{SA}$, centered on $f_0$ ($\Delta f_{SA} = f_{SA} - f_0$).

The effect of monochromatic infrared (IR) laser illumination on one of the resonance modes of the resonator is shown in inset (b) of Fig. 1, which exhibits two reflection coefficient ($|S_{11}|$) measurements which were performed near the second resonance frequency $f_0 = 3.71$ GHz and obtained using a network analyzer. The HED is biased with a sub-critical dc current of 4.14 μA, which only slightly increases the damping rate relative to the measured damping rate in the case where the HED is unbiased. The solid curve was taken without illumination whereas the dotted curve was taken while the device was being illuminated by an IR laser light having a wavelength of 1550 nm and an effective power of 27 nW. One notes that under illumination the resonance frequency substantially red shifts while the damping rate surprisingly decreases. This is caused by the fact that, as the impedance of the HED is modified, the RF current is repelled out of the HED and redistributes in the ring [29]. This measurement demonstrates frequency tuning by monochromatic IR illumination, which is characterized by the parameter $\xi Q \cong 4.1$. This measurement was repeated while gradually increasing the dc bias current, and the shift in the resonance frequency, which occurs as a response to the IR illumination, is plotted in Fig. 1. As expected, frequency shifts occurs only when the bias current $I_{bias}$ is close to the critical current, $I_c = 4.35$ μA. Note that a stable optical responsivity is achieved in the range 4.1 μA $\lesssim I_{bias} \lesssim$ 4.35 μA, whereas the system exhibits a telegraphic noise in the range 3.95 μA $\lesssim I_{bias} \lesssim$ 4.1 μA.

Fast modulation of the resonance frequency is performed using the experimental setup depicted in Fig. 2(a). The resonator is excited by a monochromatic pump tone, having a power of −50.8 dB m, at $f_{pump} = f_0$. The optical power impinging on the HED, which has an average value of 220 fW, is modulated at frequency $f_m = 2f_{pump} + \Delta f_1 \cong 7.74$ GHz, using a Mach–Zender modulator, driven by a second monochromatic signal, phase locked with the first one. The frequency offset $\Delta f_1 = 800$ Hz is chosen to be much smaller than the resonance width $f_0/Q$. Note that the laser power is approximately eight orders of magnitude lower than the power used in the experimental approach proposed by Braggio et al. [8]. Fig. 2(b) shows the reflected power off the resonator as a function of the measured spectrum analyzer (SA) frequency $f_{SA}$, centered on $f_0$ ($\Delta f_{SA} = f_{SA} - f_0$). It shows five distinguished tones. The strongest one, labeled as $f_0$, is the reflected spectral component at the frequency of the stimulating pump tone $f_{pump}$. The other four tones are found at frequencies $f_n = f_{pump} + n\Delta f_1$. For example, $f_1$ and $f_{-1}$ tones result from second ($f_1 = f_m - f_{pump}$) and fourth ($f_{-1} = 3f_{pump} - f_m$) order mixing between the power-modulated optical signal and the driving pump tone respectively. No dc bias current is employed in this measurement; instead, the pump power is tuned such that the HED is driven into a sub-critical region, close to a threshold of a nonlinear instability [32–36].

The results presented above demonstrate modulation of the resonance frequency near the frequency of the primary parametric resonance. Furthermore, the parametric gain threshold condition $\xi Q > 1$ is achieved using optical illumination having a stationary power. The main limitation, which currently prevents the achievement of parametric gain, is the relatively low photon flux that impinges the HED. We estimate that a mean number of 13 photons impinge on the HED during a single modulation period at twice the resonance frequency. When taking into account the quantum efficiency of the HED, which is probably lower than 1% [37], and its effective area, which is probably smaller than its printed area, we estimate that the optical power flux is about two orders of magnitude lower than the threshold power. This limitation occurs due to losses along the optical path, especially the expansion of the Gaussian beam from the tip of the fiber to the HED. Focusing the laser beam in future devices will overcome this problem, without increasing the total illumination power.

As was discussed above, the optical illumination employed for parametric excitation results in some undesirable heating and consequently an elevated noise. To study the noise properties of the system theoretically, we employ a perturbative model in which the contribution of damping is taken into account by adding a fictitious port, linearly coupled to the resonator. The model is valid below threshold $\xi Q < 1$, namely in the limit of small response, where dispersive and dissipative nonlinear terms can be neglected. Furthermore, as no commensurate relations exist between the resonance frequencies of different modes of our resonator, and as we apply a parametric excitation at the primary parametric resonance frequency of one of the modes, the model neglects any coupling between different resonance modes [2]. Note however that inter-mode effects can become important in the nonlinear regime, resulting in some interesting effects such as pulsing and self-oscillations [38,39].

The modes, in both the feedline and in the damping port, are assumed to be in thermal equilibrium at temperatures $T_f$ and $T_d$ respectively. Assume the case where no RF signals are injected from noisy instruments at room temperature, and the feedline is employed only for delivering the outgoing signal from the resonator. In this case, employing an ultra-low noise cryogenic amplifier, directly coupled to the feedline, may allow $T_f$ to be very close to the base temperature of the refrigerator. On the other hand, $T_d$ might be much higher due to the optical illumination. Assuming that damping in the resonator occurs mainly in the illuminated section, one may assume that $T_d$ is close to the temperature of that section. In general, however, modulating the optical power drives that section out of thermal equilibrium, thus $T_d$ should be considered as an effective temperature characterizing the non-equilibrium distribution. For the conditions appropriate for achieving parametric gain one may assume that $T_d$ is close to the critical temperature of the superconductor $T_c$.

In the case of harmonic excitation at twice the frequency of one of the resonance modes the system can act as a phase sensitive amplifier [40]. The phase dependence can be studied by employing homodyne detection, namely, by mixing the output signal, reflected off the resonator, with a local oscillator at angular frequency $\omega_0$ (the parametric excitation is at frequency $2\omega_0$) and with an adjustable phase $\phi$. We calculate the power spectrum $S(\omega, \phi)$ of the homodyne detector output, in the sub-threshold region, at angular frequency $\omega$. We find that $S(\omega, \phi)$ is periodic in $\phi$ with period $\pi$. We denote the minimum value as $S_-(\omega)$ (squeezed quadrature) and the maximum one as $S_+(\omega)$ (amplified quadrature). The derivation is similar to the one presented earlier in Ref. [41], thus we only state here the final results

$$S_\pm(\omega) = A\left\{1 - \frac{4\left(\frac{1}{Q_u} \mp \xi\right)}{Q_f\left[\left(\frac{2\omega}{\omega_0}\right)^2 + \left(\frac{1}{Q} \mp \xi\right)^2\right]}\right\} + A\frac{4}{Q_f Q_u\left[\left(\frac{2\omega}{\omega_0}\right)^2 + \left(\frac{1}{Q} \mp \xi\right)^2\right]}, \tag{1}$$

where $A = (1/2)\coth(\hbar\omega_0/2k_B T_d)$, $Q_u$ is the unloaded quality factor of the resonator, which is related to the loaded quality factor $Q$ by $1/Q = 1/Q_u + 1/Q_f$, where $Q_f$ characterizes the coupling between the resonator and the feedline.

Vacuum noise squeezing occurs when $S_- < 0.5$. Consider as an example the case where $\omega_0 = 2\pi \times 5$ GHz, $T_f = 0.01$ K, $T_d = 10$ K, $Q_f = 100$, $Q_u = 2 \times 10^4$, $\omega = 0$, and $\xi = 0.01$. Using Eq. (1) one finds $S_- = 0.2$. This example demonstrates that vacuum noise squeezing can be achieved even when $\hbar\omega_0 \ll k_B T_d$, provided that the coupling to the feedline is made sufficiently strong, that is, degradation in noise properties due to laser heating can be minimized.

In summary, we present preliminary experimental results which suggest that NbN superconducting stripline resonators may serve as an ideal tool for studying the DCE experimentally. Moreover we study theoretically the noise properties of the system and find that vacuum noise squeezing may be achieved even when the optical illumination employed for parametric excitation causes a significant local heating.

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