

Measurement of Phase and Magnitude of the Reflection Coefficient of a Quantum Dot

E. Buks, R. Schuster, M. Heiblum, D. Mahalu, V. Umansky, and H. Shtrikman

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

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We measure the phase and magnitude of the reflection coefficient of a quantum dot (QD) in the integer quantum Hall regime. This was done by coupling the QD under study to a large QD, serving as an *interferometer*, and monitoring the phase of the magnetoconductance oscillations of the coupled system. As the Coulomb blockade resonances of the QD are scanned we find two distinct and qualitatively different behaviors of the phase. Our results agree for the most part with the theoretical predictions for resonant tunneling in a noninteracting system. [S0031-9007(96)01774-7]

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According to Landauer formula the conductance of a system depends only on the absolute value squared of the transmission coefficient and therefore does not contain any phase information. However, measurement of the phase evolution of the transmission and reflection coefficients might provide added information about a mesoscopic system. An important example is the transmission, reflection, and dwell times, determined from the derivatives of the phases of the transmission and reflection coefficients with respect to the energy [1]. The phase factors can be obtained by coupling the mesoscopic system under study to an interference device which serves as an *interferometer*. This novel method had been recently employed [2] to probe the phase of the transmission coefficient of a quantum dot (QD) [3]. In this experiment a QD was inserted in one arm of an Aharonov-Bohm (AB) ring and the phase shift of the AB conductance oscillations was monitored as the resonance peaks of the QD were scanned. However, the *two terminal* nature of the measurement imposed, due to *Onsager relations*, a phase rigidity of the oscillations and prevented a direct measurement of the evolving phase in the QD [4]. Employing a *four terminal, double slit* interference structure [5] lifts this rigidity and allows a direct measurement of the phase of the transmission coefficient [6].

In the present paper we develop a new interferometry method which enables one to measure directly the magnitude and the phase of the reflection coefficient of a QD in the *integer quantum Hall* (IQH) regime. Our interference structure [see Fig. 1(a)] consists of a large circular (diameter $1.5 \mu\text{m}$) QD (the *interferometer*, on the left) coupled to a small ($0.5 \mu\text{m} \times 0.5 \mu\text{m}$) QD (the system under study, on the right). The combined structure is coupled to two 2D reservoirs, on the left (*S*) and on the right (*D*). If a magnetic field is applied perpendicularly to the plane of the two dimensional electron gas (2DEG) edge channels, associated with the intersection of the Landau levels (LL) with the Fermi level, form. Assuming that only the outer edge channel (lowest LL) in the interferometer couples out due to its close proximity to the leads, then the transmission probability through the two coupled

QD system is given by [7–9]

$$T_{DS} = \left| \frac{t_1 t_2}{1 - r_1 r_2 e^{i\varphi(B)}} \right|^2, \quad (1)$$

where t_1 and r_1 are the transmission and reflection coefficients of the small QD, while t_2 and r_2 are related to the entry quantum point contact (QPC) on the opposite side of the interferometer, and $\varphi(B)$ is the magnetic field dependent phase acquired by an electron traveling around

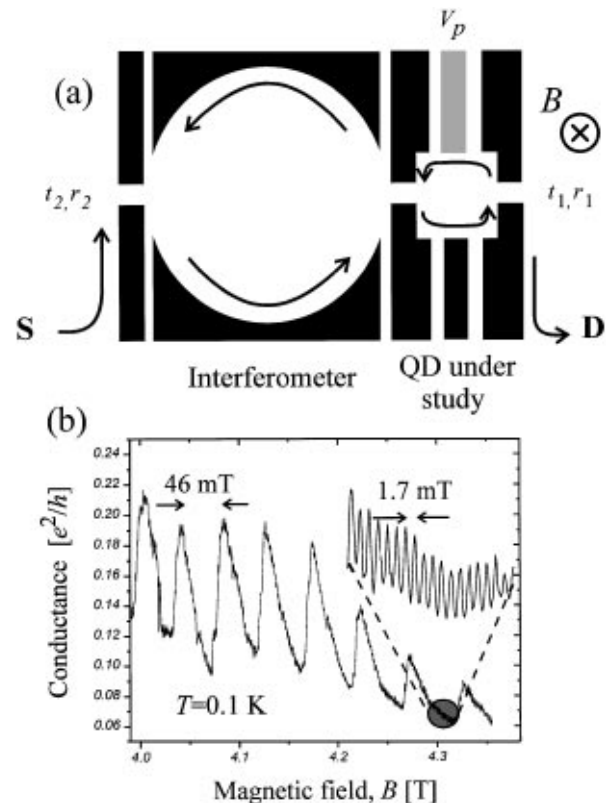


FIG. 1. (a) The interferometer on the left (large QD), coupled to a small QD on the right (the device under study). The arrows indicate the direction of current flow in the edge channels. (b) The MC is oscillating with two periods: a large period, associated with the small QD, and superimposed on it a small period, associated with the interferometer (see the enlarged regime).

the disk. Equation (1) can be derived easily by summing all the semiclassical trajectories of the outer edge channel from S to D , as they cross a multiple number of times the perimeter of the interferometer. Expanding T_{DS} in a series form yields that the n th harmonic of the magnetoconductance (MC) oscillations is proportional to $\cos n[\varphi(B) + \varphi_0]$, where $\varphi_0 = \theta(r_1) + \text{const}$, with $r_1 = |r_1|e^{i\theta(r_1)}$. This relates the phase of the reflection coefficient, $\theta(r_1)$, directly to the phase of the first harmonic of the conductance. Note that for our experimental conditions the temperature is comparable with the energy spacing in the interferometer, and, consequently, the magnitude of the MC oscillations is strongly damped; however, the phase of the oscillations is not affected. De-phasing in the interferometer might decrease further the magnitude; however, also in this case the phase is unchanged [9,10]. In general, the transmission and reflection coefficients in Eq. (1) depend on energy; however, we do not expect a significant effect due to finite temperature since both t_1 and r_1 (the small QD on the right) and t_2 and r_2 (the QPC on the left) are smooth functions of energy on the scale of the temperature [11]. The magnitude of the reflection $|r_1|$, in turn, is found from the nonoscillatory part of the conductance, given by Ohm's law: $g_0 = (e^2/h)/(|t_1|^{-2} + |t_2|^{-2})$, with $|r_1|^2 = 1 - |t_1|^2$ [10].

Interaction among electrons in a QD with an applied magnetic field results in a formation of compressible and incompressible strips [12] and a modification of the charge distribution [13]. While the derivation of Eq. (1) is based on a single particle (SP) interference approach, the existence of interaction could suggest a modified treatment. However, the many body problem can be simplified by taking into account the effect of $e-e$ interaction by replacing the external potential by an effective, self-consistent, potential, and thus reducing the problem to a SP one [14]. Although the transmission probability [Eq. (1)] has the same form as in the noninteracting case, the details of the transport are modified by the $e-e$ interaction. For example, we find experimentally that the phase $\varphi(B)$, deduced from the actual periodicity of the MC oscillations, differs significantly from the prediction of a SP model [3].

In practice, the coupled QD system was defined by metallic gates on top of a GaAs-AlGaAs heterostructure supporting a 2DEG, with a mobility $\mu = 1.6 \times 10^6$ cm²/V s and a carrier density $n_s = 3.0 \times 10^{11}$ cm⁻² at $T = 4.2$ K. Applying a negative gate voltage depletes the electrons underneath and forms the device where all QPC's that form the openings to the QD's can be adjusted individually. We allow only the outer edge channel to tunnel through the composite structure by tuning the conductance of each QPC forming the interferometer and the small QD below e^2/h . The MC was measured in a dilution refrigerator ($T_{\text{bath}} = 30$ mK, $T_{\text{electrons}} \approx 100$ mK) using standard lock-in techniques. We show below results from a device measured with a magnetic field of $B \approx 4$ T corresponding to a filling factor $\nu = 3$ in the bulk (by estimating the depletion layer width [12] we find the num-

ber of edge channels inside both QD's to be the same as in the bulk). A similar behavior is found for different devices and under conditions for different filling factors.

The MC oscillations of the double dot device are shown in Fig. 1(b). One can clearly see two different periods: a large period, associated with the small QD, and superimposed on it a small period, associated with the interferometer. The large ratio between the two periods enables us to determine φ_0 by scanning the magnetic field through several periods of MC oscillations associated with the interferometer without affecting significantly the properties of the small QD (changing the magnetic flux treating the small QD by much less than a flux quanta). Scanning the plunger gate voltage of the small QD, V_p , at a fixed magnetic field, we find pronounced peaks in the conductance indicating that the small QD is in the Coulomb blockade (CB) regime [Fig. 2(a)]. We then plot the evolution of the phase of the MC oscillations, φ_0 , found from the phase of the first harmonic of the Fourier transform of the MC data, as several CB resonances are being scanned by V_p [Fig. 2(b)]. Superimposed on a linear background, φ_0 has a periodic structure which repeats itself near each resonance.

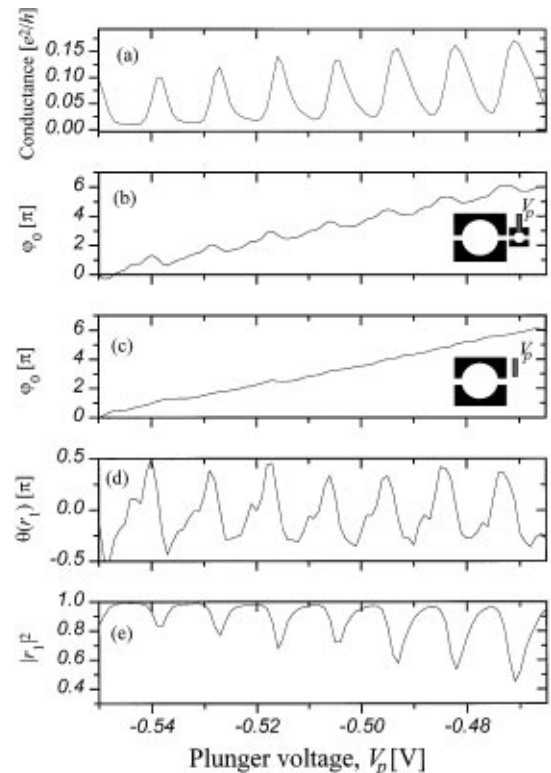


FIG. 2. (a) The CB peaks in the conductance. (b) The phase of the MC oscillations, φ_0 , of the double dot device exhibits a periodic structure with a linear background. (c) The same as in (b) but for the interferometer by itself (the small QD was removed in order to calibrate the interferometer). (d) The phase of the reflection coefficient, $\theta(r_1)$, obtained by subtracting the background calibration phase, φ_0 , in (c) from the total phase in (b). (e) The magnitude squared of the reflection coefficient, $|r_1|^2$.

As we scan the plunger voltage of the small QD, V_p , the interferometer is also affected electrostatically due to the close proximity between the two QD's, and therefore, the phase φ_0 is modified by this (undesirable) effect. To account for that we calibrate the interferometer by opening the right-most side QPC of the small QD thereby effectively eliminating the small QD. This enables a measurement of the effect of V_p on the phase acquired by an electron circulating inside the interferometer. Scanning now V_p we find small ($\approx 5\%$ of the conductance), equally spaced conductance peaks. Note that the interferometer shows conductance oscillations with the same period at zero magnetic field, indicating that these are CB oscillations (rather than AB-like oscillations, originating from a possible dependence of the interferometer area on V_p). The evolution of φ_0 is found by monitoring the MC oscillations as we vary V_p . We find [Fig. 2(c)] a linear dependence of this phase on V_p with approximately the same slope as found in the presence of the small QD; however, the periodic structure near each resonance peak disappears [15]. We determine the phase of the reflection coefficient, $\theta(r_1)$, by subtracting the background shown in Fig. 2(c) from the data in Fig. 2(b). We find [Fig. 2(d)] that $\theta(r_1)$ has an oscillatory behavior with the same period as that of the CB peaks of the small QD: it peaks with a magnitude $\leq 0.5\pi$ on the rising side of the resonance and dips with the same magnitude on the descending side of the resonance. Note the reproducible "humps" in the phase in between the CB peaks. To complete the determination of the complex coefficient r_1 , we plot in Fig. 2(e) its magnitude squared, $|r_1|^2$, found from the nonoscillatory part of the conductance.

The behavior of $\theta(r_1)$, shown in Fig. 2(d), is not unique. We retune the voltages applied to the QPC's of the small QD—attempting to form a higher barrier on the drain (D) side, and a lower barrier on the interferometer side (the reasons for doing this are explained below). We measure again the evolution of φ_0 in this regime as we scan V_p across the CB peaks seen in Fig. 3(a), and plot $\theta(r_1)$ in Fig. 3(b), after the subtraction of the linear background. The phase $\theta(r_1)$ exhibits now a totally different behavior: a monotonic rise by almost exactly 2π per CB peak, with an increased slope near the center of each peak. Note that this behavior and the one seen in Fig. 2(d) are the only ones found in our experiments.

We compare now our results with theoretical predictions of resonant tunneling through a noninteracting system. Our model system contains a scattering region with an arbitrary potential coupled (weakly) via sites R (right) and L (left) to 2 one dimensional, disorder free leads. Using a tight binding model and a standard Green function method [16] we calculate the transmission and reflection coefficients, t and r , of the system. We consider the behavior near a resonance, namely, $|E_F - E_n| \ll \Delta E$, where E_F is the Fermi energy, E_n is an eigenenergy of the isolated scattering region, and ΔE is the average energy spacing between eigenstates. Assuming a small coupling

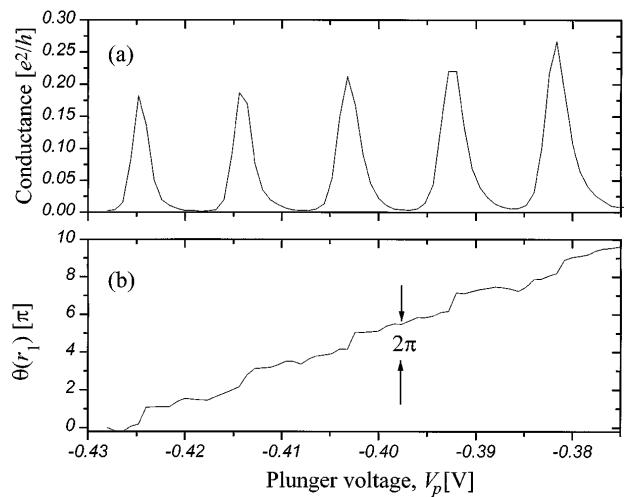


FIG. 3. (a) The CB peaks for $\gamma_- < 0$ (defined in the text). (b) The phase $\theta(r_1)$, after subtracting the linear background, increases by 2π across each resonance peak.

between the leads and the scattering region we obtain

$$t = \frac{2i\psi_R^n(\psi_L^n)^*\alpha^2}{\varepsilon - \varepsilon_n(1 + \gamma_+/2) - i\gamma_+}, \quad (2)$$

$$r = \frac{\varepsilon - \varepsilon_n(1 + \gamma_+/2) - i\gamma_-}{\varepsilon - \varepsilon_n(1 + \gamma_+/2) - i\gamma_+}, \quad (3)$$

where α is a dimensionless coupling constant between the scattering region and the leads (we assume $\alpha \ll 1$), $\psi_R^n(\psi_L^n)$ is the amplitude of the wave function of state n in the coupling site $R(L)$, $\gamma_+ = \alpha^2(|\psi_R^n|^2 + |\psi_L^n|^2)$, $\gamma_- = \alpha^2(|\psi_R^n|^2 - |\psi_L^n|^2)$; $\varepsilon = E_F/V$ and $\varepsilon_n = E_n/V$ are the normalized Fermi and eigenenergies, with $4V$ being the width of the energy band in the leads. According to Eq. (2), which is the well known Breit-Wigner formula, $|t|^2$ has a peak at $\varepsilon = \varepsilon_n(1 + \gamma_+/2)$ with a height $(\gamma_+^2 - \gamma_-^2)/\gamma_+^2$ and a width γ_+ [see Fig. 4(a)]. The phase $\theta(t)$ changes by π as ε_n is scanned across the resonance [see Fig. 4(b)]. While the peak in $|t|^2$ is associated with a dip in $|r|^2$, since $|r|^2 = 1 - |t|^2$, two different behaviors for the phase $\theta(r)$ are possible, revealing the asymmetry between the two barriers which confine the scattering region. If $\gamma_- > 0$, reflection from the side of the higher barrier, then $\theta(r)$ approaches a peak from the left of the resonance and a symmetric dip from the right of the resonance [see Fig. 4(c)]; while if $\gamma_- < 0$, reflection from the side of the lower barrier, then $\theta(r)$ changes monotonically by 2π across the resonance with an increased slope near the center of the peak [see Fig. 4(d)].

We find a good qualitative agreement between our experimental results for the behavior near the center of the peaks and the theoretical predictions: the measured $\theta(r_1)$ seen in Figs. 2(d) and 3(b) versus the calculated behavior in Figs. 4(c) and 4(d), respectively. This is somewhat surprising since $e-e$ interactions, dominating the properties

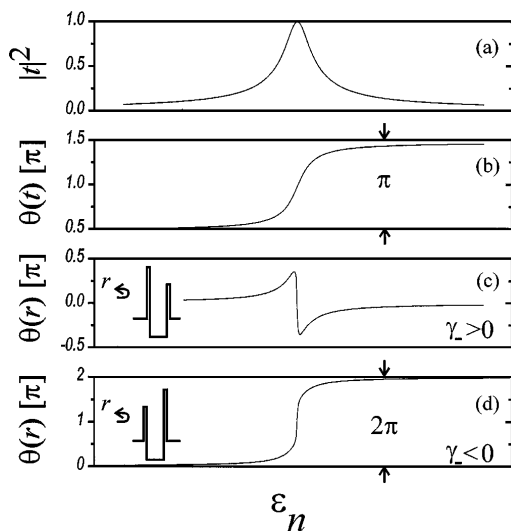


FIG. 4. Theoretical predictions for resonant tunneling in a tight binding model. (a) The transmission magnitude squared, $|t|^2$; (b) the phase of the transmission coefficient, $\theta(t)$, both calculated from Eq. (2). The phase of the reflection coefficient, $\theta(r)$: (c) for $\gamma_- > 0$ (reflection from the side of the higher barrier), and (d) for $\gamma_- < 0$ (reflection from the side of the lower barrier), calculated from Eq. (3).

of the QD, are ignored in our simple model. Note that our model is not applicable far from resonance, hence, we use the following argument for this regime. Let t_L (r_L) be the transmission (reflection) coefficient of the QPC between the small QD and the interferometer. Assuming $|t_L| \ll 1$ we find that far from resonance the total reflection of the small QD is given by $r_1 = r_L + O(|t_L|^2)$, namely, r_1 is expected to vary smoothly as a function of V_p (since the latter has a small influence on the barriers). However, a closer look at the experimental results indicates that this is not the case. We find for both cases, $\gamma_- > 0$ seen in Fig. 2(d) and $\gamma_- < 0$ seen in Fig. 3(b), small humps in the phase $\theta(r_1)$ near the points of minimum conductance. A further experimental and theoretical study is needed to clarify the origin of these humps.

We would like to compare our results to a recent *double slit* interference experiment in which the phase of the transmission coefficient of a QD was measured in zero magnetic field [6]. A good agreement is found between the experimental results of $\theta(t)$ and the theoretical prediction for the behavior near the center of the peaks, as in the present case; however, the phase $\theta(t)$ exhibits a very sharp drop by π between resonance peaks. The origin of these sharp drops is not known at the moment, leaving the behavior far from resonance, for r and t , not well understood.

In summary, we have measured the phase and the magnitude of the reflection coefficient of a QD in the IQH regime and found two distinct behaviors. The experimental results close to the resonance peaks agree with a simple theoretical model for resonant tunneling in a noninteracting system, while there are deviations away from the resonance conditions.

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