Performance of an AuPd micromechanical resonator as a temperature sensor

Ashok Kumar Pandey,[1] Oded Gottlieb,[1] Oleg Shtempluck,[2] and Eyal Buks[2,a)]
[1]Faculty of Mechanical Engineering, Technion, Haifa 32000, Israel
[2]Faculty of Electrical Engineering, Technion, Haifa 32000, Israel

(Received 12 February 2010; accepted 28 April 2010; published online 18 May 2010)

In this work we study the sensitivity of the primary resonance of an electrically excited microresonator for the possible usage of a temperature sensor. We find a relatively high normalized responsivity factor $\mathcal{R}_f = |\frac{T}{f}\frac{df}{dT}| = 0.37$ with a quality factor of $\sim 10^5$. To understand this outcome we perform a theoretical analysis based on experimental observation. We find that the dominant contribution to the responsivity comes from the temperature dependence of the tension in the beam. Subsequently, $\mathcal{R}_f$ is found to be inversely proportional to the initial tension. Corresponding to a particular temperature, the tension can be increased by applying a bias voltage. © *2010 American Institute of Physics*. [doi:10.1063/1.3431614]

Micromechanical resonators are widely used as highly sensitive sensors for sensing mass,[1] pressure,[2,3] and temperature.[4] The detection scheme of such dynamic sensors, which are commonly called resonance sensors, is based on the sensitivity of either resonance frequency[1] or quality factor[4] to changes in the measured physical quantity. In this paper, our focus is on the sensitivity of a temperature sensor based on the variation in the resonance frequency f of a fixed-fixed thin AuPd microresonator over a temperature range of 77–300 K.

In general, any detection scheme employed for monitoring the temperature T can be characterized by the ratio $\delta T/T$, where δT is the minimum detectable change in T. This ratio is determined by the averaging time τ of the measurement, and it is given by,[5,6]

$$\frac{\delta T}{T} = \mathcal{R}_X^{-1}|X_0|^{-1}\left(\frac{2\pi}{\tau}\right)^{1/2} P_X^{1/2}(0), \tag{1}$$

where $\mathcal{R}_X = |\frac{T}{X}\frac{dX}{dT}|$ is the normalized responsivity factor that characterizes the temperature dependence of the parameter X, X_0 is the average value of X, and $P_X^{1/2}(0)$ is the zero frequency spectral density of the measured parameter X. To demonstrate an important advantage of our proposed scheme over an alternative method to measure temperature we calculate below the ratio $\delta T/T$ for two cases. In the first one, we consider the most common scheme to measure temperature, in which the parameter X represents the electrical resistance R of the absorbing element. The detection of R is done by driving a current I through the absorbing element and measuring the voltage V across the device. The spectral density of V is given by Nyquist noise formula $P_V = 2Rk_BT_{\mathrm{N}}/\pi$, where k_B is the Boltzmann's constant and T_{N} is the effective noise temperature of the device. Using this result one finds that

$$\frac{\delta T}{T} = 2\mathcal{R}_R^{-1}\sqrt{\frac{k_BT_{\mathrm{N}}}{P_R\tau}}, \tag{2}$$

where $P_R = I^2R$ is the heating power due to the externally applied current. This undesirable heating factor introduces a bias in the measurement of T. However, as can be seen from Eq. (2), such a bias is unavoidable since in the presence of noise a finite sensitivity is possible only when the device is externally driven.

Similarly, for comparison we calculate below the ratio $\delta T/T$ for our proposed scheme, namely for a detector that is based on a suspended mechanical resonator. In this case the mechanical resonance frequency $f = \omega/2\pi$ is taken to be the measured temperature dependent parameter. The lowest value of the ratio $\delta T/T$ is obtained when the mechanical resonator is externally driven at its resonance frequency and its response is monitored by a homodyne detection scheme that measures the phase of oscillations. Using again Eq. (1) one finds that for this case[7]

$$\frac{\delta T}{T} = \frac{1}{Q}\mathcal{R}_f^{-1}\sqrt{\frac{k_BT_{\mathrm{N}}}{P_f\tau}}, \tag{3}$$

where $P_f = \omega U_0/Q$ is the heating power due to mechanical damping in the driven resonator, which depends on the stored mechanical energy U_0 of the driven resonator in the steady state, and on the quality factor of the resonator Q. Comparison between Eqs. (2) and (3) demonstrates an important advantage of employing resonant detection, as in our proposed scheme. For the same undesirable heating power (P_R for the first case and P_f for the second one) the sensitivity is enhanced by a factor of $1/Q$ (provided that the responsivity factors are comparable).

For a fixed-fixed resonator of length L, width B, thickness H, density ρ, and Young's modulus E, the resonance frequencies of the system under the action of a large tension N (i.e., $\beta = \sqrt{EI/NL^2} \ll 1$) can be obtained upto the second order of β using perturbation method as[8]

$$f = \frac{n}{2L}\sqrt{\frac{N}{\rho A}}\left[1 + 2\beta + \left(\frac{\pi^2n^2}{2} + 4\right)\beta^2\right] + O(\beta^3), \tag{4}$$

where I is the area moment of inertia and $A = BH$ is the cross-sectional area. Previous studies[9–11] have demonstrated that the temperature dependence of E can be exploited to yield a mechanical based temperature sensor. In this paper, on the other hand, we show that an even higher responsivity factor can be obtained by exploiting the strong temperature

a)Electronic mail: eyal@ee.technion.ac.il.

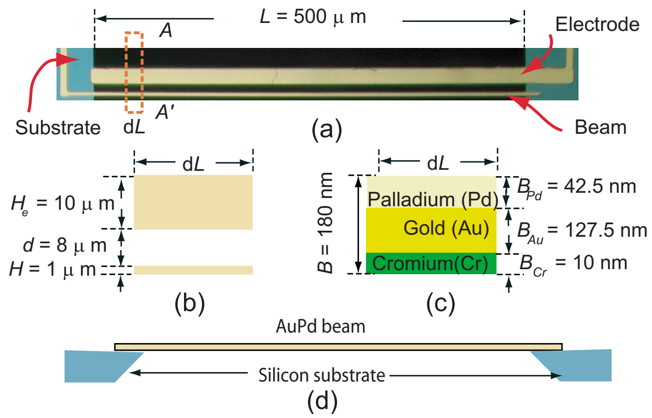

FIG. 1. (Color online) (a) A picture depicting the beam and electrode geometry on the substrate. Top view of the section dL along AA′. (c) Side view of the section AA′ across the thickness of the beam.

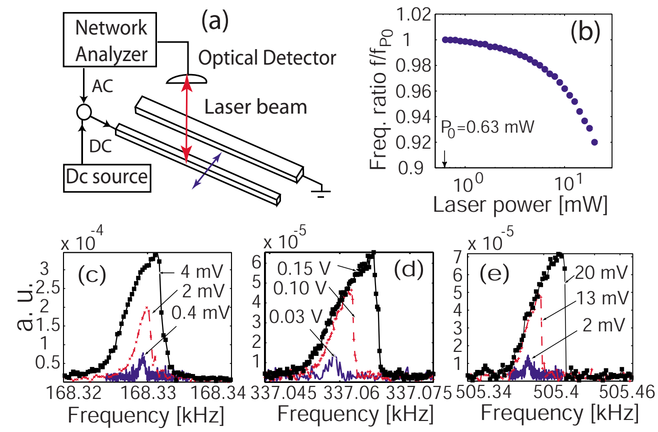

FIG. 2. (Color online) (a) Outline of the experimental setup; (b) Variation in fundamental frequency with laser power. The frequency response of the resonator at (c) the first, (d) second, and (e) the third resonant frequencies, respectively, at different ac voltages.

dependence of N in metallic doubly clamped beams. We first start with a short description of the device and the experimental setup used to characterize it at different temperatures, and then present the corresponding theoretical analysis. We end the paper with a short discussion on the temperature offset bias due to undesirable heating of the suspended beam.

Figure 1(a) depicts a doubly clamped $Au_{0.75}Pd_{0.25}$ beam on the silicon substrate of thickness 500 μm. It is fabricated using a bulk micromachining process.[8] This process also induces large residual tension in the fabricated structure due to temperature rise during the evaporation of the metal.[3] The dimensions of the fabricated composite beam as depicted in Fig. 1(b) has the following values of $L=500$ μm, $H=1$ μm, $B=180$ nm, $H_e=10$ μm, and $d=8.0$ μm. The effective Young's modulus, density, and the linear coefficient of thermal expansion are $E=99$ GPa, $\rho=16951$ kg/m^3, and $\alpha=12$ ppm/K, respectively.[12,13]

To characterize the mechanical response of the sample over the temperature range of 77–300 K, we use an optical detection principle as shown in Fig. 2(a). At the liquid nitrogen temperature, we apply 20 V dc voltage and small ac voltage to the beam, and keeping the side as well as the bottom electrodes grounded to induce the excitation of the beam. Subsequently, we find the three modes of the beam with frequencies $f_1=168.3$ kHz, $f_2=337$ kHz, and $f_3=505.4$ kHz as shown in Figs. 2(c)–2(e). The frequency ratios of the higher modes with respect to the fundamental mode are found to be 2.002 and 3.003, thus, indicating to a behavior of a taut string, i.e., $\beta \ll 1$. Additionally, the frequencies ratios are also found to be approximately the same over the entire temperature range. Consequently, the second and higher terms in Eq. (4) can be neglected. To measure the sensitivity of the sample with respect to the temperature, we heat the sample through different temperatures until 300 K and measure the corresponding fundamental frequencies as shown in Fig. 3(a). The linear approximation of the variation in frequency with temperature gives the dimensional and normalized responsivity factor, $\alpha_f=df/dT \approx 164$ Hz/K and $R_f=0.37$ at 300 K. To find the contribution of laser heating, we measure the primary resonance of the resonator at different laser powers P, viz., 0.6–20 mW as shown in Fig. 2(b). Computing the slope of the curve as $\Delta f/\Delta P \approx 0.5$ kHz/mW at around 1 mW, a temperature rise of approximately $(\Delta T/\Delta P)=(\Delta T/\Delta f)\times(\Delta f/\Delta P)\approx 3$ K/mW is found due to the laser heating.

To estimate the theoretical values of the frequency as a function of the temperature, we use the taut string model, which is obtained by neglecting the second and third terms of Eq. (4). Moreover, the tension N is expressed as $N=N_0+N_1$, where N_0 is the pretension in the fabricated device corresponding to the reference temperature $T_0=300$ K and N_1 is the additional tension induced in the beam due to the differential thermal contraction of the beam and the substrate. Based on the theory of linear thermoelasticity,[14] we can find the value of the additional tension based on the differential linear thermal coefficients from

$$N_1 = -EA(\alpha-\alpha_s)(T-T_0), \tag{5}$$

where E is the Young's modulus, and α and α_s are the linear coefficients of thermal expansion of the AuPd beam and Silicon substrate, respectively (here, for silicon $\alpha_s=2.6$ ppm/K). Substituting the values of the physical

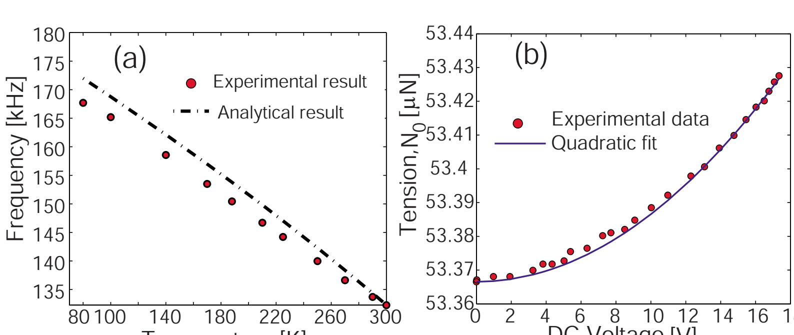

FIG. 3. (Color online) (a) Primary resonance of the beam under external excitation at temperature varies from 80 to 300 K; (b) Variation in tension with dc voltage at 300 K.

properties, and taking the experimental value of the fundamental frequency at reference temperature T_0, we get $N_0 = 53.3\ \mu\text{N}$ from Eq. (5). Considering the properties E, α, and α_s as temperature independent and using $N_0=53.3\ \mu\text{N}$, we estimate the theoretical value of the frequency as shown in Fig. 3(a). The percentage error over the temperature range of 80–300 K is found to be less than 3%. Note that the only fitting parameter is the pretension N_0.

Using the above relations, the corresponding normalized responsivity factor at $T=T_0$ is obtained from $\mathcal{R}_f \approx -(T_0/2N_0)EA(\alpha-\alpha_s)$. From this relation, $\mathcal{R}_f$ is also found to be proportional to N_0^{-1}. Hence, the sensitivity can also be tuned by changing the initial tension N_0 by a bias voltage as shown in Fig. 3(b). This effect is attributed to the elongation of the resonator.[15] In addition to the responsivity factor, Eq. (3) suggests that sensitivity enhancement can be achieved by increasing the net stored energy U_0, namely by driving the resonator more strongly. However, in practice this may result in undesirable heating that in turn can lead to a temperature rise of the driven beam-string. Furthermore, a similar undesirable heating may occur due to the electrical heating. However, because of a relatively high quality factor and large capacitive impedance of the device, the heating due to the above mentioned sources are found to be negligible as compared to that due to the laser heating. However, in our analysis it is also to be noted that the substrate's temperature is assumed to be fixed and the effective thermal fluctuations[6] in the temperature is disregarded. Another important figure of merit is the response time corresponding to the change in resonance frequency of the suspended beam due to a sudden change in the temperature of the substrate. This time is characterized by the thermal time constant $\tau_T=(\rho c/\kappa)(L^2/\pi^2)$, where c and κ are the effective specific heat and thermal conductivity, respectively.[16] For the given resonator, it is found to be 2.6 ms. Another important factor that gives rise to finite response time is the ring-down time t_{RD} of the mechanical resonator, which is given by $t_{\text{RD}}=Q/f \sim 1-2$ s. Due to the relatively high Q value of our resonator, this time scale is much longer than τ_T.

To discuss the thermal noise, it can be seen from Eq. (3) that the temperature spectral density S_{T} of the sensor depends on the effective noise temperature T_{N}, which in turn, is given by $T_{\text{N}}=T(1+S_{\text{x,DT}}/S_{\text{x,TM}})$, where $S_{\text{x,DT}}$ is the displacement spectral density of the optical detection scheme and $S_{\text{x,TM}} = 2Qk_BT/[\pi\rho AL(2\pi f)^3]$ is the resonance value of the displacement spectral density of thermomechanical noise.[6] For our setup the main contribution to $S_{\text{x,DT}}$ comes from current noise in the photodetector. Assuming injected optical power of $P_{\text{O}}=10^{-4}$ W one finds that $S_{\text{x,DT}}^{1/2}=2.5\times 10^{-12}\ \text{m}/\sqrt{\text{Hz}}$.[17] Moreover, for the fundamental mechanical mode of our device $S_{\text{x,TM}}^{1/2}=7\times 10^{-12}\ \text{m}/\sqrt{\text{Hz}}$, thus $T_{\text{N}}/T \simeq 1.1$. Using these estimates and a typical value for mechanical damping heating power of $P_f=10^{-14}$ W yield $S_{\text{T}}^{1/2} \simeq 10^{-6}\ \text{K}/\sqrt{\text{Hz}}$.

In conclusion, we propose a temperature sensor based on the sensitivity of the induced tension of an AuPd microresonator near its primary resonance. The tension is found to be induced due to the differential thermal expansion coefficients of the resonator and the substrate. A relatively high normalized responsivity factor $R_f=0.37$ is found with a low thermal spectral density of $1\ \mu\text{K}/\sqrt{\text{Hz}}$.

We thank S. Zaitsev for his help in fabrication of the microresonator. A.K.P. also thanks Lady Davis Foundation, Technion, for the Postdoctoral Fellowship. This work is partially supported by the Russell Berrie Nanotechnology Institute, the Germany Israel Foundation, the Israel Science Foundation (ISF 1043/07), and the European STREP project QNEMS.

[1]K. L. Ekinci, X. M. H. Huang, and M. L. Roukes, Appl. Phys. Lett. **84**, 4469 (2004).
[2]S. S. Verbridge, H. G. Craighead, and J. M. Parpia, Appl. Phys. Lett. **92**, 013112 (2008).
[3]X. M. H. Huang, M. Manolidis, S. C. Jun, and J. Hone, Appl. Phys. Lett. **86**, 143104 (2005).
[4]M. A. Hopcroft, B. Kim, S. Chandorkar, R. Melamud, M. Agarwal, C. M. Jha, G. Bahl, J. Salvia, H. Mehta, H. K. Lee, R. N. Candler, and T. W. Kenny, Appl. Phys. Lett. **91**, 013505 (2007).
[5]A. N. Cleland, New J. Phys. **7**, 235 (2005).
[6]K. L. Ekinci, Y. T. Yang, and M. L. Roukes, J. Appl. Phys. **95**, 2682 (2004).
[7]E. Buks and B. Yurke, Phys. Rev. E **74**, 046619 (2006).
[8]E. Buks and M. L. Roukes, Phys. Rev. B **63**, 033402 (2001).
[9]A. B. Hutchinson, P. A. Truitt, K. C. Schwab, L. Sekaric, J. M. Parpia, H. G. Craighead, and J. E. Butler, Appl. Phys. Lett. **84**, 972 (2004).
[10]J. Gaspar, V. Chu, and J. P. Conde, Appl. Phys. Lett. **84**, 622 (2004).
[11]R. Sandberg, W. Svendsen, K. Molhave, and A. Boisen, J. Micromech. Microeng. **15**, 1454 (2005).
[12]http://en.wikipedia.org/wiki/Gold, Palladium, and Chromium.
[13]F. Shen, P. Lu, S. J. O'Shea, K. H. Lee, and T. Y. Ng, Sens. Actuators, A **95**, 17 (2001).
[14]L. D. Landau and E. M. Lifshitz, *Theory of Elasticity*, 3rd ed. (Oxford, New York, 1986).
[15]C. Chen, S. Rosenblatt, K. I. Bolotin, W. Kalb, P. Kim, I. Kymissis, H. L. Stormer, T. F. Heinz, and J. Hone, Nat. Nanotechnol. **4**, 861 (2009).
[16]S. C. Jun, X. M. H. Huang, M. Manolidis, C. A. Zorman, M. Mehregany, and J. Hone, Nanotechnology **17**, 1506 (2006).
[17]D. Karabacak, T. Kouh, C. C. Huang, and K. L. Ekinci, Appl. Phys. Lett. **88**, 193122 (2006).