Hysteresis and intermittency in direct-current superconducting quantum interference device with nanobridges fabricated on a thin membrane

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This study examined the voltage response of nanobridge-based direct-current superconducting quantum interference devices (dc-SQUIDs) fabricated on a Si_3N_4 membrane. Such a configuration may help in reducing $1/f$ noise, which possibly originates from substrate fluctuating defects. Results showed that the poor thermal coupling between the dc-SQUID and the substrate leads to a strong hysteretic response of the superconducting quantum interference device (SQUID) even though it is biased by an alternating current. In addition, when the dc-SQUID is biased near a threshold of spontaneous oscillations, the measured voltage has an intermittent pattern, which depends on the applied magnetic flux threading the loop of the SQUID. © *2011 American Institute of Physics*. [doi:10.1063/1.3549767]

Superconducting quantum interference devices (SQUIDs) are key components in many applications.[1] One of the major goals of current research on SQUIDs is to reduce noise in the Josephson Junctions (JJs) composing the SQUIDs. Previous studies have shown that one of the main sources of $1/f$ noise is the coupling of the JJs to fluctuating two-level systems either in the substrate[2] or on oxidized metal surfaces and interfaces.[3] Several studies with single-electron transistors (SETs) have tried to solve this problem by suspending the junctions composing the SETs in order to decouple them from the substrate. To date, these studies have only shown moderate success.[4–6] Another emerging technology attracting increasing interest is that of SQUIDs based on nanobridge weak-links. Such SQUIDs can be made extremely small and have the potential to outperform conventional SQUIDs in terms of noise properties.[7,8]

Our original research goal was to study the noise properties of a nanobridge-based dc-SQUID that is fabricated on a suspended silicon nitride (Si_3N_4) membrane. However, we discovered that nonequilibrium thermal processes that emerge due to the poor coupling of the JJs to the substrate play a dominant role in SQUID dynamics. A strong hysteretic response of the dc-SQUID is observed when the SQUID is excited by alternating current even though such excitation should, in principle, eliminate hysteretic behavior. In addition, when the SQUID is biased near a threshold of spontaneous oscillations, the measured voltage shows an intermittent pattern that depends on the applied magnetic flux threading the loop of the SQUID. In order to account for these results, we extended the theoretical model in Ref. 9 to include low-frequency variations in the SQUID temperature. Numerical simulations show qualitative agreement with the experimental data.

A simplified circuit layout of a typical device is illustrated in Fig. 1(b). Our devices are fabricated on a high-resistivity silicon wafer covered by a 100 nm thick layer of Si_3N_4. A small section of the wafer is etched from the back to produce a suspended Si_3N_4 membrane in the size of $100 \times 100\ \mu m^2$. The dc-SQUID is made of a 80 nm thick niobium and has lateral dimensions of $110 \times 7\ \mu m^2$. Using the FastHenry program,[10] the self-inductance of the SQUID is calculated to be $L = 100\ pH$. The dc-SQUID is composed of two nanobridge JJs (NBJJs) with one NBJJ in each of its two arms. The area of the bridges is $100 \times 115\ nm^2$, and their combined critical current is 1.2 mA. Most of the SQUID structure including one of the NBJJs is fabricated on the Si_3N_4 membrane, as shown by the electron micrographs in Figs. 1(c) and 1(d). An on-chip stripline passes near the dc-SQUID and is used to generate a magnetic flux in the dc-SQUID loop. A dc bias line that includes on-chip low-pass filters (LPFs) is connected to the dc-SQUID and is used for voltage measurements of the SQUID. Note that although the dc-SQUID is embedded in a superconducting stripline resonator [not illustrated in Fig. 1(b)], its influence on the experiments presented in this paper is expected to be negligible. Further design considerations and fabrication details can be found elsewhere.[11,12]

Our experiments were carried out using the setup depicted in Fig. 1(a). We used a lock-in amplifier, which ap-

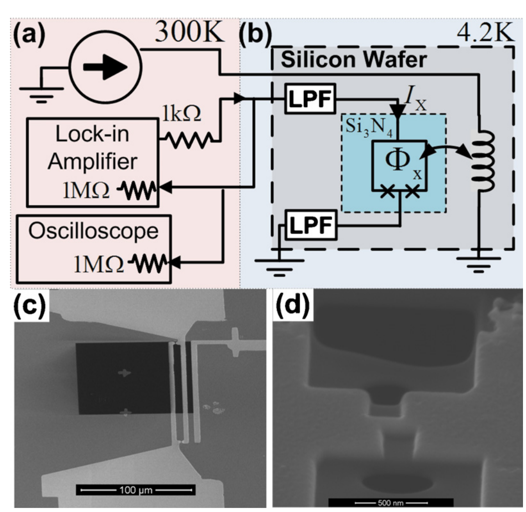

FIG. 1. (Color online) (a) Measurement setup. (b) A simplified circuit layout for a dc-SQUID. (c) Electron micrograph of a dc-SQUID partially fabricated on a Si_3N_4 membrane. (c) Electron micrograph image of a nanobridge.

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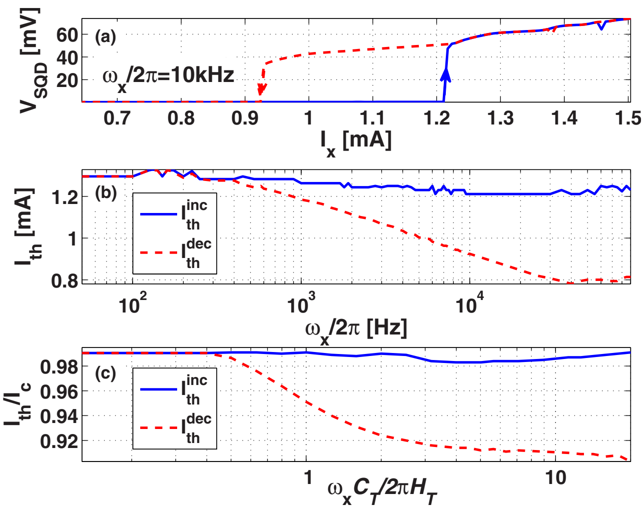

FIG. 2. (Color online) Hysteretic current-voltage measurements. Panel (a) shows dc-SQUID voltage traces measured for increasing and decreasing current excitation sweeps. Panels (b) and (c) show measurement and simulation results, respectively, of increasing and decreasing threshold currents as a function of the excitation frequency.

plies alternating current through the SQUID, with frequencies up to 100 kHz. We measured the voltage across the dc-SQUID with the lock-in amplifier and recorded its time domain dynamics using an oscilloscope. In experiments showing intermittency, we also applied dc magnetic flux threading the loop of the SQUID. All measurements were carried out while the device was fully immersed in liquid helium.

Figure 2(a) shows an example of the dc-SQUID hysteretic response to an excitation current with an alternating frequency of $\omega_x/2\pi=10$ kHz. This measurement shows the measured voltage across the SQUID when the current amplitude is swept up and then down. Each recorded point is averaged over 100 ms, thereby including 1000 excitation cycles. Such alternating excitation should, in principle, eliminate hysteresis in the response of the dc-SQUID, provided that all characteristic time-scales of the device are much shorter than the excitation period. Note that the thermal relaxation rate of a local hot-spot, created in a nanobridge with good thermal coupling to the substrate, is typically on the order of gigahertz.[13] However, measuring the characteristic behavior of hysteretic current-voltage under alternating current excitation reveals that there is a much longer characteristic thermal time-scale, on the order of milliseconds, which affects the dc-SQUID dynamics. Such a long time-scale may exist in our device due to the relatively large heat capacity of the Si_3N_4 membrane to which the SQUID is coupled as well as due to the poor coupling between this membrane and its surroundings, which results in a relatively long thermal relaxation time.

Figure 2(b) summarizes several measurements as in Fig. 2(a). It plots the increasing and decreasing threshold currents (i.e., the values of applied currents corresponding to voltage jumps in the increasing and decreasing current sweeps, respectively) as a function of the lock-in amplifier carrier frequency. These threshold currents are approximately equal only up to frequencies of about 500 Hz. The hysteretic nature of the dc-SQUID emerges when the excitation frequency increases even further. The decreasing threshold current falls to lower values while the increasing one only slightly decreases. This trend continues up to frequencies of about 40 kHz, when both threshold currents cease to depend on the excitation frequency.

In order to account for the above results, we extended the model in Ref. 9, which describes the metastability of dc-SQUIDs and considers variations in the SQUID temperature. The equations of motion for the NBJJ phases γ_1 and γ_2 are given by

$$\ddot{\gamma}_1+\beta_D\dot{\gamma}_1+(1+\alpha_0)y(\Theta)\sin\gamma_1+\frac{1}{\beta_{L0}}(\gamma_1-\gamma_2+2\pi\Phi_x/\Phi_0)=I_x/I_{c0}+g_n, \quad (1)$$

$$\ddot{\gamma}_2+\beta_D\dot{\gamma}_2+(1-\alpha_0)y(\Theta)\sin\gamma_2-\frac{1}{\beta_{L0}}(\gamma_1-\gamma_2+2\pi\Phi_x/\Phi_0)=I_x/I_{c0}+g_n, \quad (2)$$

where the overdot denotes a derivative with respect to a normalized time parameter $\tau=\omega_{pl}t$ and where ω_{pl} is the dc-SQUID plasma frequency. In addition, I_x is the bias current, Φ_x is the external magnetic flux, Φ_0 is flux quantum, β_D is the dimensionless damping coefficient, and E_0 is the Josephson energy. The critical current of the dc-SQUID at the temperature of the coolant T_0 is denoted by I_{c0}. The dimensionless parameters β_{L0} and α_0 characterize the dc-SQUID hysteresis and asymmetry at temperature T_0, respectively. The dimensionless factor g_n is a noise term, which is neglected in our numerical calculations.

The term $y(\Theta)$ expresses the dependence of the NBJJs' critical currents on the temperature. It is given by $y(\Theta)\equiv\tilde{y}(\Theta)/\tilde{y}(\Theta_0)$,[14] where $\Theta=T/T_c$, $\Theta_0=T_0/T_c$, and T_c is the critical temperature of the dc-SQUID. The function $\tilde{y}$ is given by $\tilde{y}(\Theta)=(1-\Theta^2)^{3/2}(1+\Theta^2)^{1/2}$. The spatial dependence of the temperature is disregarded. Using the notation $\beta_C=2\pi C_T T_c/\Phi_0 I_{c0}$ and $\beta_H=H_T/C_T\omega_{pl}$, where C_T is thermal heat capacity and H_T is heat transfer rate, the SQUID heat balance equation reads

$$\dot{\Theta}=\frac{\beta_D}{\beta_C}(\dot{\gamma}_1^2+\dot{\gamma}_2^2)-\beta_H(\Theta-\Theta_0). \quad (3)$$

Simulation results showing the dependence of the hysteretic behavior of the dc-SQUID on the excitation frequency are shown in Fig. 2(c). This panel shows the increasing and decreasing threshold currents as a function of the excitation frequency, which are normalized by the thermal relaxation time. These results show qualitative agreement with the corresponding experimental results shown in Fig. 2(b).

Figure 3 shows the intermittent behavior of the SQUID. In this measurement, the dc-SQUID is biased near the threshold of spontaneous oscillations using alternating current at a frequency of $\omega_x/2\pi=70$ Hz. The voltage across the SQUID is measured as a function of time for various values of magnetic flux threading the loop of the SQUID. Panel (a) shows the average value of the squared voltage versus the magnetic flux. Panels (b)–(e) show voltage time traces measured for the corresponding points marked in panel (a). The low-amplitude voltage oscillations appear due to the existence of a parasitic serial resistance in the experimental wiring setup. Each time the alternating current drives the SQUID into the oscillatory zone, it responds with a voltage spike that is measured on top of the low-amplitude parasitic voltage. Spikes

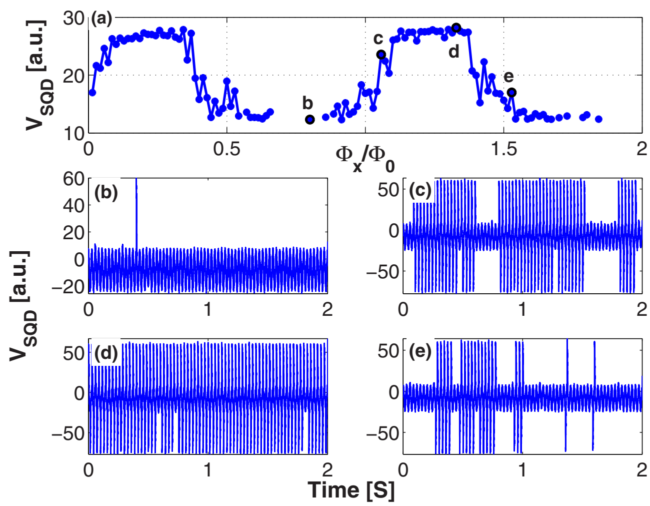

FIG. 3. (Color online) Intermittent behavior. (a) Measured average squared voltage of the dc-SQUID. (b)–(e) Time traces of the dc-SQUID voltage taken at the working points marked by the corresponding letter in panel (a).

can be either positive or negative depending on the polarity of the driving current. The measured traces show intermittent behavior in which the spikes are generated in bunches, suggesting that in this region, the system becomes thermally bistable. This bistability is realized by switching between the cold locally stable state, where the dc-SQUID is in a superconducting state, and the hot locally stable state, where the SQUID becomes normal. Switching between these states is randomly triggered by external noise.

In conclusion, this study examined nanobridge-based dc-SQUIDs fabricated on a Si_3N_4 membrane. Results showed that the response of a dc-SQUID to a low-frequency alternating excitation current is hysteretic and that the strength of the hysteresis depends on the excitation frequency. In addition, when the SQUID is biased near a threshold of spontaneous oscillations, the measured voltage has an intermittent pattern that depends on the applied magnetic flux threading the loop of the SQUID. Such a hysteretic response degrades the performance of the suspended SQUID and limits the effectiveness of using suspension as a method for noise reduction, at least for SQUIDs with relatively large critical currents.

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