Damping in a Superconducting Mechanical Resonator

Oren Suchoi[1] and Eyal Buks[1]

[1] *Andrew and Erna Viterbi Department of Electrical Engineering, Technion, Haifa 32000 Israel*

PACS 74.25.Ld – Superconductivity , Mechanical and acoustical properties, elasticity, and ultrasonic attenuation
PACS 46.40.- f –v[redacted]brations and mechanical waves

Abstract – We study a mechanical resonator made of aluminum near the normal to super conductivity phase transition. A sharp drop in the rate of mechanical damping is observed below the critical temperature. The experimental results are compared with predictions based on the Bardeen Cooper Schrieffer theory of superconductivity and a fair agreement is obtained.

Mechanical resonators having low damping rate are widely employed for sensing and timing applications [1]. At sufficiently low temperatures such devices may allow the experimental exploration of the crossover from classical to quantum mechanics [2–8]. Commonly, the observation of non-classical effects in such experiments is possible only when the damping rate [9] of the mechanical resonator is sufficiently low. Mechanical resonators made of superconductors are widely employed in such low-temperature experiments. In this study we experimentally investigate the effect of superconductivity on the damping rate of a mechanical resonator made of aluminum near its normal to super phase transition.

The damping rate can be measured by coupling the mechanical resonator under study to a displacement detector. In general, a variety of different mechanisms may contribute to the total mechanical damping rate. In order to isolate the effect of superconductivity it is important to employ a method of displacement detection that is unaffected by the normal to super phase transition. In addition, systematic errors in the measured damping rate due to back-reaction effects originating from the coupling between the mechanical resonator and the displacement detector have to be kept at a sufficiently low level.

In our setup (see Fig. 1) we employ the so-called optomechanical cavity configuration [10–12], in which displacement detection is performed by coupling the mechanical resonator to an electromagnetic cavity. While many of the previous studies of superconducting mechanical resonators have employed such a configuration with a superconducting microwave cavity [4, 6, 7, 13–21], our setup, which is based on a cavity in the optical band, allows displacement detection that is unaffected by the phase transition occurring in the mechanical resonator under study, which, in-turn, allows isolating the effect of superconductivity on the mechanical damping. Moreover, back-reaction effects are suppressed by employing a relatively low driving power to the optical cavity (see discussion below).

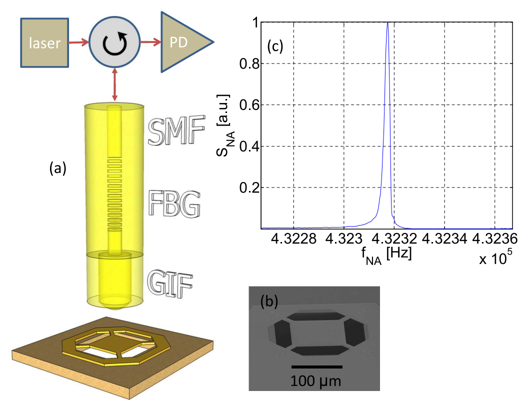

Fig. 1: The experimental setup. (a) A sketch of the mechanical resonator and the fiber-based optical cavity. (b) Electron micrograph of the trampoline. (c) The resonance lineshape of the fundamental mechanical mode.

In our setup the optomechanical cavity is formed between two mirrors, a stationary fiber Bragg grating (FBG) mirror and a movable mirror made of a mechanical resonator in the shape of a trampoline supported by four beams [see Fig. 1(a)]. A graded index fiber (GIF) spliced to the end of the single mode fiber (SMF) is employed for focusing. A cryogenic piezoelectric three-axis positioning system having sub-nanometer resolution is employed for manipulating the position of the optical fiber. A photolithography process is used to pattern a $t_{\mathrm{Al}} = 200\,\mathrm{nm}$ thick aluminum layer on top of a a $t_{\mathrm{SiN}} = 100\,\mathrm{nm}$ thick silicon nitride layer into the shape of a $100 \times 100\,\mu\mathrm{m}^2$ trampoline with four supporting beams [see Fig. 1(b)]. Details of the fabrication process can be found elsewhere [22]. Measurements are performed in a dilution refrigerator at a pressure well below $2 \times 10^{-5}\,\mathrm{mbar}$. Another device on the same wafer, which is made in the shape of a microwave microstrip resonator, allows characterizing the surface resistance of the aluminum layer [22].

A tunable laser operating near the Bragg wavelength of the FBG together with an external attenuator are employed to excite the optical cavity. The optical power reflected off the cavity is measured by a photodetector (PD), which is connected to a network analyzer (NA). Actuation is performed by applying an alternating voltage (with a direct voltage offset) between the trampoline and a stationary electrode positioned $200\,\mu\mathrm{m}$ below it.

The fundamental mechanical mode is characterized by its frequency $\omega_{\mathrm{m}}/2\pi$ and damping rate γ_{m}. Both parameters can be extracted from the resonance lineshape of the measured NA signal S_{NA} vs. angular driving frequency ω_{NA} with a fixed driving amplitude [see Fig. 1(c)]. In the regime of linear response $S_{\mathrm{NA}}(\omega_{\mathrm{NA}})$ is expected to be given by

$$S_{\mathrm{NA}}(\omega_{\mathrm{NA}}) = \frac{S_{\mathrm{NA,R}}}{1 + \left(\frac{\omega_{\mathrm{NA}} - \omega_{\mathrm{m}}}{\gamma_{\mathrm{m}}}\right)^2}, \tag{1}$$

where $S_{\mathrm{NA,R}}$ is the value of $S_{\mathrm{NA}}(\omega_{\mathrm{NA}})$ at resonance, i.e. when $\omega_{\mathrm{NA}} = \omega_{\mathrm{m}}$.

The measured damping rate γ_{m} vs. temperature T, which is extracted from the NA data using Eq. (1), is indicated by the crosses in Fig. 2. The same procedure yields the mode's frequency $\omega_{\mathrm{m}}/2\pi = 432.318\,\mathrm{kHz}$, which is found to be almost a constant for the range of temperatures explored in this measurement (between $0.5\,\mathrm{K}$ and $1.3\,\mathrm{K}$). As can be seen from Fig. 2, the measured damping rate γ_{m} sharply drops as the temperature T is lowered below the value of $1.1\,\mathrm{K}$. Note that at the same temperature of $1.1\,\mathrm{K}$ the resonance of the microwave microstrip resonator becomes visible, indicating thus that the critical temperature is $T_{\mathrm{c}} = 1.1\,\mathrm{K}$, as is expected from high quality aluminum layer. The data seen in Fig. 2 is obtained by setting the laser power that is injected into the optical cavity to the value $P_{\mathrm{L}} = 3 \times 10^{-8}\,\mathrm{W}$.

In general, heating due to optical absorption by the aluminum layer may give rise to a systematic error in the measurement of γ_{m}. Two possible mechanisms are discussed below. The first one is due to back-reaction originating from the bolometric optomechanical coupling [23,24],

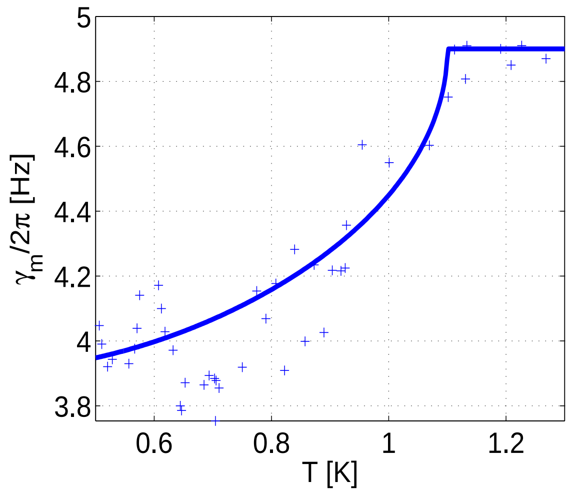

Fig. 2: The measured (crosses) and theoretically calculated (solid line) damping rate γ_{m} vs. temperature T. The calculated γ_{m} is obtained using Eq. (2).

which gives rise to a shift in the effective value of γ_{m} denoted by $\gamma_{\mathrm{m,ba}}$. The magnitude of $\gamma_{\mathrm{m,ba}}$ can be roughly estimated using the relation $|\gamma_{\mathrm{m,ba}}|/\gamma_{\mathrm{m}} \simeq P_{\mathrm{L}}/P_{\mathrm{LT}}$, where P_{L} is the laser power that is employed for the measurement of γ_{m} and P_{LT} is the laser power at the threshold of self-excited oscillation [25]. For the same cavity tuning, for which the data seen in Fig. 2 is obtained, self-excited oscillation occurs at a threshold power given by $P_{\mathrm{LT}} = 4 \times 10^5 P_{\mathrm{L}}$, and thus the effect of back-reaction can be safely disregarded.

The other possible source of a systematic error originates from temperature rise due to optical absorption. Due to the low heat conductance of both superconducting aluminum and silicon nitride, this mechanism imposes a severe upper limit upon the allowed values of laser power. The heating power is given by $P_{\mathrm{H}} = \zeta\beta_{\mathrm{F}}(1 - R_{\mathrm{C}})P_{\mathrm{L}}$, where ζ is the absorption coefficient (for aluminum $\zeta = 0.03$), β_{F} is the cavity finesse, R_{C} is the cavity reflectivity, and P_{L} is the laser power [26]. For the measurement of γ_{m} that is presented in Fig. 2 the optical cavity is tuned to have finesse of $\beta_{\mathrm{F}} = 1.8$ and reflectivity of $R_{\mathrm{C}} = 0.32$, and thus for this measurement $P_{\mathrm{H}} = 1.1 \times 10^{-9}\,\mathrm{W}$.

The temperature rise ΔT due to optical absorption is estimated by $\Delta T = P_{\mathrm{H}}/4K_{\mathrm{b}}$, where K_{b} is the thermal conductance of each of the four nominally identical beams that support the trampoline. The thermal conductance K_{b} is given by $K_{\mathrm{b}} = (t_{\mathrm{Al}}\kappa_{\mathrm{Al}} + t_{\mathrm{SiN}}\kappa_{\mathrm{SiN}})(w_{\mathrm{b}}/l_{\mathrm{b}})$, where κ_{Al} (κ_{SiN}) is the thermal conductivity of aluminum (silicon nitride), and where $w_{\mathrm{b}}/l_{\mathrm{b}} = 0.5$ is the ratio between width and length of the beams. For the lowest value of $0.5\,\mathrm{K}$ in the temperature range seen in Fig. 2 the thermal conductivities are estimated to be $\kappa_{\mathrm{Al}} \simeq 4 \times 10^{-2}\,\mathrm{W\,K^{-1}\,m^{-1}}$ for aluminum [27] and $\kappa_{\mathrm{SiN}} \simeq 10^{-2}\,\mathrm{W\,K^{-1}\,m^{-1}}$ for silicon nitride [28,29]. For these values the estimated temperature rise is $\Delta T = 0.06\,\mathrm{K}$. For temperatures below 0.5 K the temperature rise ΔT becomes even larger since both κ_{Al} and κ_{SiN} rapidly drops at low temperatures, and therefore no reliable measurements can be obtained unless P_{H} is further reduced. However, no significant reduction of P_{H} is possible in our setup due to noise, and consequently reliable data far below 0.5 K cannot be obtained. Note, however, that above 0.5 K no significant change in the damping rate is obtained when the measurements are repeated with a laser power two times higher, verifying thus that heating does not give rise to a significant systematic error in that range.

To account for the experimental results the measured damping rate γ_{m} is compared with theory. The solid line seen in Fig. 2 represents the calculated value of γ_{m} obtained from the following expression [30, 31]

$$\gamma_{\mathrm{m}} = \gamma_{\mathrm{S}} + \frac{2\gamma_{\mathrm{N}}}{1 + \exp \frac{\Delta(T)}{k_{\mathrm{B}} T}}, \tag{2}$$

where the fitting parameters γ_{S}, γ_{N} and T_{c} are taken to be given by $\gamma_{\mathrm{S}}/2\pi = 3.9\,\mathrm{Hz}$, $\gamma_{\mathrm{N}}/2\pi = 1.0\,\mathrm{Hz}$ and $T_{\mathrm{c}} = 1.1\,\mathrm{K}$. The temperature dependent energy gap $\Delta(T)$ is found by numerically solving the Bardeen Cooper Schrieffer (BCS) gap equation [32]

$$\nu = \int_0^{\frac{e^{\nu}}{2\delta}} \mathrm{d}x \, \frac{\tanh\left(\frac{\xi\delta\sqrt{1+x^2}}{\tau}\right)}{\sqrt{1+x^2}} , \tag{3}$$

where $\nu = 1/gD_0$ is the inverse interaction strength with g being the electron-phonon coupling coefficient and D_0 being the density of states per unit volume, $\delta = \Delta/\Delta_0$ is the normalized gap with Δ_0 being the zero temperature gap, $\tau = T/T_{\mathrm{c}}$ is the normalized temperature, and the number ξ is given by $\xi = \pi/2e^{C_{\mathrm{E}}}$ with $C_{\mathrm{E}} \simeq 0.577$ being the Euler's constant. As can be seen from Fig. 2, fair agreement between data and theory is obtained. A similar theoretical approach has been employed before to successfully account for the results of a measurement of a contact-less friction between a superconducting niobium film and a cantilever [33] (see also [34]).

The rate γ_{N} represents the contribution of normal electrons to the total rate of mechanical damping [see Eq. (2)]. This rate has been calculated in Ref. [35] for the case where the electron mean-free path is greater than the wavelength of the oscillating acoustic mode. However, this assumption is not valid for our device. When the electron mean-free path is shorter than the acoustic wavelength the rate γ_{N} can be roughly estimated using Stokes' law of sound attenuation [36, 37], which relates this rate to the electronic viscosity [38]. For the case of an acoustic wave having angular frequency ω_{m} propagating in a bulk aluminum the rate according to this approach is given by

$$\gamma_{\mathrm{N,bulk}} = \frac{2\eta_{\mathrm{Al}}\omega_{\mathrm{m}}^2}{3\rho_{\mathrm{Al}} c_{\mathrm{Al}}^2}, \tag{4}$$

where $\rho_{\mathrm{Al}} = 2.7\,\mathrm{g\,cm^{-3}}$ and $c_{\mathrm{Al}} = 5.1 \times 10^3\,\mathrm{m\,s^{-1}}$ are the mass density and the speed of sound, respectively, of aluminum, and where η_{Al} is the electronic viscosity of aluminum. The electronic viscosity can be expressed as $\eta_{\mathrm{Al}} = (2/3)\, n_{\mathrm{Al}} \tau_{\mathrm{Al}} \langle \epsilon_{\mathrm{Al}} \rangle$ where n_{Al} is the density of free electrons, τ_{Al} is the scattering relaxation time, and $\langle \epsilon_{\mathrm{Al}} \rangle$ is the averaged kinetic energy [see Eq. (43.8) in [39]]. At low temperatures $\langle \epsilon_{\mathrm{Al}} \rangle = 3\epsilon_{\mathrm{F,Al}}/5$, where $\epsilon_{\mathrm{F,Al}}$ is the Fermi energy. Using the values $n_{\mathrm{Al}} = 18.1 \times 10^{22}\,\mathrm{cm^{-3}}$, $\tau_{\mathrm{Al}} = 6.5 \times 10^{-14}\,\mathrm{s}$ and $\epsilon_{\mathrm{F,Al}} = 11.7\,\mathrm{eV}$ [40] one obtains $\gamma_{\mathrm{N,bulk}} = 0.6\,\mathrm{Hz}$. This rough estimate, which disregards confinement, yields a value that is of the same order of magnitude as the value of $\gamma_{\mathrm{N}}/2\pi = 1.0\,\mathrm{Hz}$ that has been obtained above from the fitting of the data with Eq. (2).

In summary, the contribution of free electrons to the damping rate of an aluminum mechanical resonator is measured. The behavior near the normal to super phase transition is compared with a prediction based on the BCS theory. The relative contribution of free electrons in the normal state to the total damping rate of the resonator under study is found to be $\gamma_{\mathrm{N}}/(\gamma_{\mathrm{N}} + \gamma_{\mathrm{S}}) = 0.20$.

This work was supported by the Israel Science Foundation, the Binational Science Foundation, the Security Research Foundation at Technion and the Russell Berrie Nanotechnology Institute.

REFERENCES

[1] K.L. Ekinci and M.L. Roukes, "Nanoelectromechanical systems", *Rev. Sci. Inst.*, vol. 76, pp. 061101, 2005.

[2] Pierre Meystre, "A short walk through quantum optomechanics", *Annalen der Physik*, vol. 525, no. 3, pp. 215–233, 2013.

[3] M. Poot and H. S.J. van der Zant, "Mechanical systems in the quantum regime", *Phys. Rep.*, vol. 511, pp. 273–335, 2012.

[4] A. D. OConnell, M. Hofheinz, M. Ansmann, Radoslaw C. Bialczak, M. Lenander, Erik Luceroand M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, John M. Martinis, and A. N. Cleland, "Quantum ground state and single-phonon control of a mechanical resonator", *Nature*, vol. 464, pp. 697–703, 2010.

[5] Igor Pikovski, Michael R Vanner, Markus Aspelmeyer, MS Kim, and Časlav Brukner, "Probing planck-scale physics with quantum optics", *Nature Physics*, vol. 8, no. 5, pp. 393–397, 2012.

[6] A. J. Weinstein, C. U. Lei, E. E. Wollman, J. Suh, A. Metelmann, A. A. Clerk, and K. C. Schwab, "Observation and interpretation of motional sideband asymmetry in a quantum electromechanical device", *Phys. Rev. X*, vol. 4, pp. 041003, Oct 2014.

[7] JD Teufel, Tobias Donner, Dale Li, JW Harlow, MS Allman, Katarina Cicak, AJ Sirois, Jed D Whittaker, KW Lehnert, and Raymond W Simmonds, "Sideband cooling of micromechanical motion to the quantum ground state", *Nature*, vol. 475, no. 7356, pp. 359–363, 2011.

[8] Jasper Chan, TP Mayer Alegre, Amir H Safavi-Naeini, Jeff T Hill, Alex Krause, Simon Gröblacher, Markus Aspelmeyer, and Oskar Painter, "Laser cooling of a nanome-

chanical oscillator into its quantum ground state", *Nature*, vol. 478, no. 7367, pp. 89–92, 2011.

[9] Quirin P Unterreithmeier, Thomas Faust, and Jörg P Kotthaus, "Damping of nanomechanical resonators", *Physical review letters*, vol. 105, no. 2, pp. 027205, 2010.

[10] V. B. Braginsky and A. B. Manukin, "Ponderomotive effects of electromagnetic radiation (in Russian)", *ZhETF (Journal of Experimental and Theoretical Physics)*, vol. 52, pp. 986–989, 1967, Sov. Phys. JETP 25, 653 (1967).

[11] Markus Aspelmeyer, Tobias J Kippenberg, and Florian Marquardt, "Cavity optomechanics", *Reviews of Modern Physics*, vol. 86, no. 4, pp. 1391, 2014.

[12] Michael Metcalfe, "Applications of cavity optomechanics", *Applied Physics Reviews*, vol. 1, no. 3, pp. 031105, 2014.

[13] JD Teufel, Dale Li, MS Allman, K Cicak, AJ Sirois, JD Whittaker, and RW Simmonds, "Circuit cavity electromechanics in the strong-coupling regime", *Nature*, vol. 471, no. 7337, pp. 204–208, 2011.

[14] TA Palomaki, JD Teufel, RW Simmonds, and KW Lehnert, "Entangling mechanical motion with microwave fields", *Science*, vol. 342, no. 6159, pp. 710–713, 2013.

[15] S Yanai, V Singh, M Yuan, M Gely, SJ Bosman, and GA Steele, "Mechanical dissipation in more superconducting metal drums", *arXiv:1608.01871*, 2016.

[16] RW Andrews, AP Reed, K Cicak, JD Teufel, and KW Lehnert, "Quantum-enabled temporal and spectral mode conversion of microwave signals", *Nature Communications*, vol. 6, pp. 10021, 2015.

[17] F. Lecocq, J. B. Clark, R. W. Simmonds, J. Aumentado, and J. D. Teufel, "Mechanically mediated microwave frequency conversion in the quantum regime", *Phys. Rev. Lett.*, vol. 116, pp. 043601, Jan 2016.

[18] J. Suh, M. D. Shaw, H. G. LeDuc, A. J. Weinstein, and K. C. Schwab, "Thermally induced parametric instability in a back-action evading measurement of a micromechanical quadrature near the zero-point level", *Nano Lett.*, vol. 12, pp. 6260–6265, 2012.

[19] E. E. Wollman, C. U. Lei, A. J. Weinstein, J. Suh, A. Kronwald, F. Marquardt, A. A. Clerk, and K. C. Schwab, "Quantum squeezing of motion in a mechanical resonator", *Science*, vol. 349, pp. 952–955, 2015.

[20] J.-M. Pirkkalainen, E. Damskägg, M. Brandt, F. Massel, and M. A. Sillanpää, "Squeezing of quantum noise of motion in a micromechanical resonator", *Phys. Rev. Lett.*, vol. 115, pp. 243601, Dec 2015.

[21] Oren Suchoi, Keren Shlomi, Lior Ella, and Eyal Buks, "Time-resolved phase-space tomography of an optomechanical cavity", *Physical Review A*, vol. 91, no. 4, pp. 043829, 2015.

[22] Oren Suchoi, Lior Ella, Oleg Shtempluk, and Eyal Buks, "Intermittency in an optomechanical cavity near a subcritical hopf bifurcation", *Physical Review A*, vol. 90, no. 3, pp. 033818, 2014.

[23] C. Metzger, M. Ludwig, C. Neuenhahn, A. Ortlieb, I. Favero, K. Karrai, and F. Marquardt, "Self-induced oscillations in an optomechanical system driven by bolometric backaction", *Phys. Rev. Lett.*, vol. 101, pp. 133903, Sep 2008.

[24] Stav Zaitsev, Ashok K. Pandey, Oleg Shtempluck, and Eyal Buks, "Forced and self-excited oscillations of an optomechanical cavity", *Phys. Rev. E*, vol. 84, pp. 046605, Oct 2011.

[25] Yuvaraj Dhayalan, Ilya Baskin, Keren Shlomi, and Eyal Buks, "Phase space distribution near the self-excited oscillation threshold", *Phys. Rev. Lett.*, vol. 112, pp. 210403, May 2014.

[26] Keren Shlomi, D Yuvaraj, Ilya Baskin, Oren Suchoi, Roni Winik, and Eyal Buks, "Synchronization in an optomechanical cavity", *Physical Review E*, vol. 91, no. 3, pp. 032910, 2015.

[27] AV Feshchenko, O-P Saira, and JP Pekola, "Thermal conductance of nb thin films at sub-kelvin temperatures", *arXiv:1609.06519*, 2016.

[28] BL Zink and F Hellman, "Specific heat and thermal conductivity of low-stress amorphous si–n membranes", *Solid State Communications*, vol. 129, no. 3, pp. 199–204, 2004.

[29] Adam L Woodcraft, Rashmi V Sudiwala, Elley Wakui, Ravinder S Bhatia, James J Bock, and Anthony D Turner, "Thermal conductance measurements of a silicon nitride membrane at low temperatures", *Physica B: Condensed Matter*, vol. 284, pp. 1968–1969, 2000.

[30] BNJ Persson, "Electronic friction on a superconductor surface", *Solid state communications*, vol. 115, no. 3, pp. 145–148, 2000.

[31] Lewis T Claiborne Jr and Robert W Morse, "Study of the attenuation of ultrasonic shear waves in superconducting aluminum", *Physical Review*, vol. 136, no. 4A, pp. A893, 1964.

[32] John Bardeen, Leon N Cooper, and John Robert Schrieffer, "Theory of superconductivity", *Physical Review*, vol. 108, no. 5, pp. 1175, 1957.

[33] Marcin Kisiel, Enrico Gnecco, Urs Gysin, Laurent Marot, Simon Rast, and Ernst Meyer, "Suppression of electronic friction on nb films in the superconducting state", *Nature materials*, vol. 10, no. 2, pp. 119–122, 2011.

[34] A Dayo, W Alnasrallah, and J Krim, "Superconductivity-dependent sliding friction", *Physical Review Letters*, vol. 80, no. 8, pp. 1690, 1998.

[35] Ze'ev Lindenfeld and Ron Lifshitz, "Damping of mechanical vibrations by free electrons in metallic nanoresonators", *Physical Review B*, vol. 87, no. 8, pp. 085448, 2013.

[36] GG Stokes, "On the theories of the internal friction of fluids in motion, and of the equilibrium and motion of elastic fluids", *Transactions of the Cambridge Philosophical Society*, vol. 8, pp. 287, 1845.

[37] Charles Kittel, "Ultrasonics research and the properties of matter", *Reports on Progress in Physics*, vol. 11, no. 1, pp. 205, 1947.

[38] Martin Heinisch, Thomas Voglhuber-Brunnmaier, Erwin K Reichel, Isabelle Dufour, and Bernhard Jakoby, "Reduced order models for resonant viscosity and mass density sensors", *Sensors and Actuators A: Physical*, vol. 220, pp. 76–84, 2014.

[39] Charles Kittel, *Elementary statistical physics*, Courier Corporation, 2004.

[40] Neil W. Ashcroft and N. David Mermin, *Solid State Physics*, Saunders College, Philadelphia, c1976.