Mode locking in an optomechanical cavity

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We experimentally study a fiber-based optical ring cavity integrated with a mechanical resonator mirror and an optical amplifier. The device exhibits a variety of intriguing nonlinear effects including synchronization and self-excited oscillation. Passively generated optical pulses are observed when the frequency of the optical ring cavity is tuned very close to the mechanical frequency of the suspended mirror. The optical power at the threshold of this process of mechanical mode locking is found to be related to quantum noise of the optical amplifier.

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Optomechanical cavities [1-8] are widely employed for various sensing applications. The effect of radiation pressure typically governs the optomechanical coupling (i.e. the coupling between the electromagnetic cavity and the mechanical resonator that serves as a movable mirror) when the finesse of the optical cavity is sufficiently high, whereas, bolometric effects can contribute to the optomechanical coupling when optical absorption by the vibrating mirror is significant [4, 9–17].

Here we study laser mode locking [19] in an optomechanical cavity. Some applications for optomechanical cavities with integrated gain medium have been proposed before, including cooling [20], squeezing of noise [21–23], controlling dynamical instabilities [24] and normal mode splitting [25]. Moreover, optomechanical cavities driven by externally injected pulses have been studied in [26–28]. Optical pulses generated by externally modulating cavity length have been discussed in [29–31]. In the current study we explore a new method of passive mode locking, which is based on bolometric optomechanical coupling.

The optical ring cavity (ORC) is schematically depicted by Fig. 1. Optical gain is generated by a Cband Erbium-doped fiber optical amplifier (OA), and loss in the ORC is controlled by adjusting the direct voltage applied to the electro-optical amplitude modulator (EOAM). The state of polarization is controlled using two polarization controllers (PCs) and a fiber polarizer (FP). A fiber Bragg grating (FBG) [32] provides optical filtering. The optical signal is detected by a photodetector (PD) connected to a radio frequency filter (RFF) and a radio frequency amplifier (RFA), which can generate a feedback signal applied to the radio frequency port of the EOAM. Note that feedback signal is applied to the EOAM only for performing the measurements presented in Fig. 2 below.

The effect of modulation (either by the moving mirror or by the EOAM) on the state of the ORC is discussed below. In the limit of small modulation amplitude the effect of noise cannot be disregarded, as was shown in [33]. The ORC optical intensity $L_{\rm H}(t)$ is expressed as $L_{\rm H}(t) = L_0 \mathcal{V}(\omega_{\rm R} t)$, where L_0 is the average intensity, $\omega_{\rm R}$ is the spacing between angular frequencies of the ORC modes, t is time and the function $\mathcal{V}(s)$ is

1% PD RFF RFA 99% FC PC 2 50 2 3 FBG FBG FBG

FIG. 1: The experimental setup. Red lines represent single mode optical fibers, whereas black lines represent radio frequency transmission lines (coaxial cables). The lump elements integrated into the ring cavity are (in counter clockwise direction, starting from the top circulator): manual polarization controller (PC), fiber coupler (FC), optical amplifier (OA), circulator connected to a fiber Bragg grating (FBG), electro-optical amplitude modulator (EOAM), electrically controlled PC and a fiber polarizer (FP). The 1% output port of the FC is connected to a photodetector (PD). The output PD signal is connected to a radio frequency filter (RFF) and a radio frequency amplifier (RFA), and the RFA output signal can be used as a feedback signal feeding the EOAM. The single mode fiber is terminated near the mechanical mirror by an FBG (having a negligible effect, since its filtering band does not overlap the one of the other FBG, that is integrated on the left arm of the ORC) and a graded index fiber (GIF) serving as a lens with a focal distance of $40 \,\mu\text{m}$. Details of the fabrication process used for preparing the suspended $100 \,\mu\text{m} \times 100 \,\mu\text{m}$ mechanical mirror made of aluminum can be found in [18]. All measurements are performed at temperature of 77 K and pressure well below 2×10^{-5} mbar.

expressed as a sum over all contributing modes of the ORC $\mathcal{V}(s) = N_{\mathrm{R}}^{-1} \left\langle \left| \sum_{m} r_{m} e^{i(ms+\theta_{m})} \right|^{2} \right\rangle$, where N_{R} is the number of ORC modes within the FBG filtering band, and the positive r_{m} and the real θ_{m} are the amplitude and phase, respectively, of the *m*'th ORC mode.

Consider an amplitude modulation applied to the ORC. When fluctuations in modes' amplitudes r_m can be disregarded the evolution of the phases θ_m is governed by a set of coupled Langevin equations given by [33]

$$\dot{\theta}_m = \mu_{\mathcal{M}} \left(\sin \left(\theta_{m-1} - \theta_m \right) + \sin \left(\theta_{m+1} - \theta_m \right) \right) + q_m ,$$
(1)

where the terms proportional to the modulation amplitude $\mu_{\rm M}$ represent the contribution of modulationgenerated sidebands of neighboring modes, and the terms q_m represent white noise satisfying correlation relations given by $\langle q_{m'}(t') q_{m''}^*(t'') \rangle = 2T_{\rm N} \delta_{m'm''} \delta(t' - t'')$, where $T_{\rm N}$ is a constant. The main source of noise in the current experiment is quantum noise of the OA [34]. In term of the Hamiltonian $\mathcal{H}(\{\theta_m\})$, which is given by $\mathcal{H} = -\mu_{\rm M} \sum_m \cos(\theta_{m-1} - \theta_m)$, Eq. (1) can be expressed as $\dot{\theta}_m = -\partial \mathcal{H}/\partial \theta_m + q_m$. In steady state the probability distribution $\mathcal{P}(\{\theta_m\})$ is given by $\mathcal{P} = Z^{-1}e^{-\mathcal{H}/T_{\rm N}}$, where Z is the partition function [35]. In the limit of weak noise the following holds $\left\langle (\theta_{m-1} - \theta_m)^2 \right\rangle = 2\beta_{\rm N}$, where $\beta_{\rm N} = T_{\rm N}/2\mu_{\rm M}$, and thus the phase correlation function is given by $\left\langle e^{i(\theta_{m-k}-\theta_m)^2} \right\rangle = e^{-|k|\beta_{\rm N}}$. Using these results one finds that $\mathcal{V}(s) = \mathcal{T}_{\beta_{\rm N}}(s)$ in the limit of weak modulation amplitude $N_{\rm R}^{-1} \ll \beta_{\rm N}$, where the nicknamed comb function $\mathcal{T}_{\beta}(s)$, which is given by

$$\mathcal{T}_{\beta}(s) = \sum_{k=-\infty}^{\infty} e^{iks - |k|\beta} = \frac{\sinh\beta}{\cosh\beta - \cos s} , \qquad (2)$$

represents a periodic train of pulses having linewidth given by $\beta/2 + O(\beta^2)$, and the averaged value of $\mathcal{T}_{\beta}(s)$ is unity for any given β .

In the other extreme of relatively large modulation amplitude $N_{\rm R}\beta_{\rm N} \ll 1$ optical noise can be approximately disregarded. Consider a Gaussian optical pulse [36] having amplitude $E(t) = E_0 \exp\left(-\gamma t^2 + i\omega_{\rm p} t\right)$ circulating inside the ORC, where E_0 is a complex constant, $\gamma = \gamma' + i\gamma''$, $\gamma' = \operatorname{Re} \gamma > 0$ determines the width of the pulse, $\gamma'' = \operatorname{Im} \gamma$ represents a linear chirp, the real $\omega_{\rm p}$ is the optical angular frequency and t is time. The effect of each of the lump elements integrated into the ORC on the mode shape is characterized by either a time-like or a frequency-like Möbius transformation $\gamma_{\text{out}}^{-1} = (A\gamma_{\text{in}}^{-1} + B) / (C\gamma_{\text{in}}^{-1} + D)$ [37]. For a frequency-like transformation characterized by the parameter γ_{F} one has $\gamma_{\text{out}}^{-1} = \gamma_{\text{in}}^{-1} + \gamma_{\text{F}}^{-1}$, i.e. $B = \gamma_{\text{F}}^{-1}$ and C = 0, whereas for a time-like transformation characterized by the parameter $\gamma_{\rm T}$ one has $\gamma_{\rm out} = \gamma_{\rm in} + \gamma_{\rm T}$, i.e. B = 0and $C = \gamma_{\rm T}$. For both cases A = D = 1. Concatenating a time-like transformation with parameter $\gamma_{\rm T}$, and a frequency-like transformation with parameter $\gamma_{\rm F}$ yields a Möbius transformation with coefficients $A = 1 + g_{\rm m}^2$, $B = \gamma_{\rm F}^{-1}$, $C = \gamma_{\rm F} g_{\rm m}^2$ and D = 1, where $g_{\rm m} = \left(\gamma_{\rm T} \gamma_{\rm F}^{-1}\right)^{1/2}$.

Two cases of mode locking are discussed below, one is based on the EOAM and the other on the moving mirror. The effect of both elements is characterized by a time-like transformation with a parameter $\gamma_{\rm T}$. Both OA and FBG optical filter give rise to a frequency-like transformation. The magnitude $|\gamma_{\rm F}|$ of the parameter $\gamma_{\rm F}$ can be expressed in terms of an effective optical wavelength band $\Delta \lambda$ using the relation $|\gamma_{\rm F}|^{1/2} = ((2\pi c) / (\lambda_{\rm L} n_{\rm eff})) (\Delta \lambda / \lambda_{\rm L})$, where c is the speed of light in vacuum, $n_{\text{eff}} = 1.47$ is the fiber mode effective refractive index and $\lambda_{\rm L} = 1550 \,\rm nm$ is the optical wavelength. For the OA in the current experiment $\Delta \lambda \simeq 50$ nm, whereas for the FBG $\Delta \lambda \simeq 0.2$ nm, and thus to a good approximation the transformation parameter $\gamma_{\rm F}$ can be evaluated by disregarding the effect of the OA. For both methods of mode locking that are employed in the current experiment the dimensionless parameter $|g_{\rm m}|$ is at most about 10⁻⁵. When $|g_{\rm m}| \ll 1$ it is convenient to represent the discrete Möbius transformation by a continuous differential equation of the normalized pulse parameter $g = \gamma / \gamma_{\rm F}$. To lowest nonvanishing order in g and $g_{\rm m}$ one finds that g evolves according to $dg/d\tau_{\rm R} = g_{\rm m}^2 - g^2$, where $\tau_{\rm R} = t/t_{\rm R}$, and $t_{\rm R}$ is the ORC period time. Out of the two fixed points $\pm g_{\rm m}$, only the one having a positive real value is stable. The solution is given by $g = g_{\rm m} \tanh (g_{\rm m} (\tau_{\rm R} - \tau_{\rm R0}))$, where $\tau_{\rm R0}$ is a constant.

Mode locking based on the EOAM is explored without integrating the mechanical mirror. This is done by placing the optical fiber (which is connected to a piezoelectric positioner) above a pad near the trampoline. The pad, which is made of the same aluminum and silicon nitride layers (as the suspended mirror), serves as a static mirror. In this mode of operation active mode locking can be induced by driving the EOAM at a frequency $\omega_{\rm AM}/2\pi$ close to the ORC frequency $\omega_{\rm R}/2\pi = 371.3$ kHz. In addition pulses can be obtained by the method of regenerative mode locking (RGML) [38, 39], in which the PD signal generates a feedback signal driving the EOAM (see Fig. 1). The phase of the pulses generated by RGML can be locked by simultaneously driving the EOAM at a frequency $\omega_{\rm AM}/2\pi$ close to the ORC frequency $\omega_{\rm R}/2\pi$. This is demonstrated in Fig. 2(a), which shows a color-coded plot of the spectral density (measured using a spectrum analyzer) of the PD signal as a function of the voltage amplitude $V_{\rm AM}$ of a modulation signal at a fixed frequency of $\omega_{\rm AM}/2\pi = 371.4 \, \rm kHz = \omega_{\rm R}/2\pi + 100 \, \rm Hz$ applied to the EOAM. Synchronization occurs in the region $V_{\rm AM} \geq V_{\rm AM,0} = 0.156 \, \text{V}$. In the unlocked region, multiple side bands emerge via a continuous transition. Such side band structure is consistent with the pulse remaining synchronized with the modulation for extended periods of time, interlaced by rapid "phase slip" events. Unlocking can be well approximated by solving the equation of motion for the relative phase $\varphi_{\rm S}$ between the pulsing oscillation and the applied modulation, given by [40, 41]

$$\frac{\mathrm{d}\varphi_{\mathrm{S}}}{\mathrm{d}\tau} + \sin\varphi_{\mathrm{S}} = i_{\mathrm{b}} , \qquad (3)$$

where $i_{\rm b} = (\omega_{\rm AM} - \omega_{\rm R}) / (\zeta_{\rm AM} V_{\rm AM})$ is a normalized detuning, the coefficient $\zeta_{\rm AM}$ is given by $\zeta_{\rm AM} = (\omega_{\rm AM} - \omega_{\rm R}) / V_{\rm AM,0}$ (i.e. $i_{\rm b} = 1$ at the onset of synchronization) and $\tau = \zeta_{\rm AM} V_{\rm AM} t$ is a dimensionless time vari-

able. The solution of Eq. (3) in the region $|i_{\rm b}| > 1$ can be expressed in terms of the comb function \mathcal{T}_{β} that is defined by Eq. (2) as $d\varphi_{\rm S}/d\tau = \sinh\beta_{\rm b}\mathcal{T}_{\beta_{\rm b}}(\tau \sinh\beta_{\rm b}+\theta_{\rm b})$, where $\beta_{\rm b} = \cosh^{-1}i_{\rm b}$ and the phase $\theta_{\rm b}$ is given by $\theta_{\rm b} = \pi - \tan^{-1}(\sinh\beta_{\rm b})$ [41]. The resulting spectrum is shown in Fig. 2(b).

The effect of the mechanical mirror is explored by studying two dynamical instabilities, self-excited oscillation (SEO) (see Fig. 3) and mechanical mode locking (MML) (see Fig. 4). For studying SEO the ORC frequency $\omega_{\rm R}/2\pi = 2.48$ MHz is not tuned close to the mechanical frequency $\omega_{\rm m}/2\pi = 415$ kHz, whereas the MML measurements are performed after adjusting the total length $L_{\rm R}$ of the ORC to satisfy the condition $\omega_{\rm R} \simeq \omega_{\rm m}$. As was mentioned above, for both cases no feedback signal is applied to the EOAM.

Let L_{SC} be the total length of the optical cavity that is formed between the fiber's tip and the mirror. This cavity is henceforth referred to as the short cavity (SC). to avoid confusion with the much longer fiber ORC. The length $L_{\rm SC}$ can be controlled by adjusting the voltage V_z that is applied to one of the piezoelectric motors moving the fiber. The plot shown in Fig. 3(a) presents the measured averaged PD voltage $V_{\rm PD}$ as a function of V_z . The two local minima points of $V_{\rm PD}$ (obtained with $V_z = 2 \, V$ and $V_z = 51 \text{ V}$, respectively) represent two optical resonances of the SC (i.e. the SC length $L_{\rm SC}$ is shortened by $\lambda_{\rm L}/2$ by increasing the voltage from the value $V_z = 2 \,{\rm V}$ to the value $V_z = 51$ V). The spectral density of the PD signal (measured using a spectrum analyzer) is shown in Fig. 3(b). The intense spectral peak that is observed in the regions $V_z \in [11 \text{ V}, 26 \text{ V}]$ and $V_z \in [56 \text{ V}, 70 \text{ V}]$ occurs due to mechanical SEO.

The response of the system is next explored in the region where $\omega_{\rm R} \simeq \omega_{\rm m}$ [$\omega_{\rm R}$ is tuned to this value by adjusting the length $L_{\rm R}$ of the ORC to the value $(c/n_{\rm eff})/(\omega_{\rm m}/2\pi) = 553.88 \,{\rm m}$, where $\omega_{\rm m}/2\pi =$ 368.2 kHz for the mechanical mirror used for these measurements]. The accuracy of this procedure is typically about 0.01 Hz. The measured averaged PD voltage $V_{\rm PD}$ as a function of V_z is shown in Fig. 4(a). The color-coded plot in Fig. 4(b) shows time traces measured by an oscilloscope connected to the PD. The time traces presented in Fig. 4(c-e) are obtained for the values $V_z = 1.2 \text{ V}$, $V_z = 17.2 \,\mathrm{V}$ and $V_z = 51.5 \,\mathrm{V}$, respectively [these values are indicated by overlaid white dotted lines in Fig. 4(b)]. The pulses shown in Fig. 4 are attributed to mirror motion that effectively generates modulation at the frequency of the mechanical oscillation. From the measured linewidth of the pulses and the relation $g_{\rm m} = \left(\gamma_{\rm T} \gamma_{\rm F}^{-1}\right)^{1/2}$ one finds that the modulation amplitude $|\gamma_{\rm T}|^{1/2} \simeq 4 \,{\rm MHz}$ for the narrowest peaks seen in Fig. 4. Note that, contrary to the case of SEO that is observed with red detuning (see Fig 3), the MML shown in Fig. 4 is obtained mainly with blue detuning.

Both effects of SEO and MML are attributed to bolometric optomechanical coupling. Consider an optical



FIG. 2: Synchronization of RGML. (a) The spectral density of the PD signal as a function of the EOAM modulation amplitude $V_{\rm AM}$. (b) The calculated spectral density is obtained from the solution of Eq. (3).

cavity with a movable mirror having mass $m_{\rm m}$, intrinsic mechanical angular frequency $\omega_{\rm m}$ and an intrinsic mechanical damping rate $\gamma_{\rm m} \ll \omega_{\rm m}$. It is assumed that the angular resonance frequency of the mechanical resonator depends on the temperature $T_{\rm m}$ of the suspended mirror. For small deviation $T_{\rm R} = T_{\rm m} - T_{\rm b}$ of $T_{\rm m}$ from the base temperature $T_{\rm b}$ (i.e. the temperature of the supporting substrate) it is taken to be given by $\omega_{\rm m} + \Theta_{\rm PH} T_{\rm R}$, where $\Theta_{\rm PH}$ is a constant. Furthermore, to model the effect of thermal deformation [11] it is assumed that a temperature dependent force given by $F_{\rm T} = \Theta_{\rm FH} T_{\rm R}$, where $\Theta_{\rm FH}$ is a constant, acts on the mechanical resonator. The mechanical oscillator's equation of motion is given by $\ddot{x}_{\rm m} + 2\gamma_{\rm m}\dot{x}_{\rm m} + (\omega_{\rm m} + \Theta_{\rm PH}\tilde{T}_{\rm R})^2 x_{\rm m} = m_{\rm m}^{-1}F_{\rm T},$ where an overdot denotes differentiation with respect to time. Intrinsic mechanical nonlinearities of the mirror are disregarded, i.e. it is assumed that nonlinear behavior exclusively originates from bolometric optomechanical coupling.

The time evolution of the relative temperature $T_{\rm R}$ is governed by the thermal balance equation $\dot{T}_{\rm R} = H_{\rm m} - \kappa_{\rm m}T_{\rm R}$, where $H_{\rm m}$ is proportional to the optically-induced heating power and $\kappa_{\rm m} \simeq 0.01\omega_{\rm m}$ is the thermal decay rate. The relative phase between heating $H_{\rm m}$ and relative temperature $T_{\rm R}$ for a steady state solution of the thermal balance equation is $\theta_{\rm T} - \pi/2$, where $\theta_{\rm T} = \tan^{-1}\kappa_{\rm m}/\omega_{\rm m}$. The heating term $H_{\rm m}$ (which is proportional to the intracavity optical power incident on the suspended mirror) is expressed as $H_{\rm m} = L_{\rm H} (t) A_{\rm H} (x_{\rm m})$, where $L_{\rm H}$ is the optical intensity. To second order in the mechanical displacement $x_{\rm m}$ the optical absorption coefficient of the mirror $A_{\rm H}$ is expressed as $A_{\rm H} = A_{\rm H0} (1 + k_{\rm A1}x_{\rm m} + k_{\rm A2}x_{\rm m}^2) + O(x_{\rm m}^3)$. The dependency of $A_{\rm H}$ on the mechanical displacement $x_{\rm m}$ originates from interference in the SC



FIG. 3: Self-excited oscillation. (a) The averaged PD voltage $V_{\rm PD}$ as a function of V_z . (b) The spectral density of the PD signal (in normalized dB units).



FIG. 4: Mechanical mode locking. (a) The averaged PD voltage $V_{\rm PD}$ as a function of the voltage V_z applied to the piezoelectric motor. (b) Oscilloscope time traces of the PD signal. The values of V_z corresponding to the time traces presented in (c), (d) and (e) are indicated by the three overlaid vertical white dotted lines in (b).



FIG. 5: Relative phase of the thermal force $F_{\rm T}$. (a) Sketch of the mirror optical absorption coefficient $A_{\rm H}$ as a function of mechanical displacement $x_{\rm m}$ near an optical resonance. The absorption coefficient k_{A1} is negative (positive) when the cavity is blue (red) detuned. In plots (b) and (c) the overlaid dots on the circles schematically represent the following oscillating variables: mechanical displacement $x_{\rm m}$, mechanical velocity $v_{\rm m}$, optically-induced heating $H_{\rm m}$, pulsing P, relative temperature $T_{\rm R}$ and thermal force $F_{\rm T}$. The relative angles between dots represent the corresponding relative phase between the oscillating variables. The plot in (b) represents the contribution of continuous wave (CW) optical intensity, whereas the contribution of pulsing is described by the plot in (c). For both cases the relative phase between mechanical displacement $x_{\rm m}$ and heating $H_{\rm m}$ depends on the detuning of the short cavity. The assumed detuning is indicated by coloring the letters $H_{\rm m}$, P, $T_{\rm R}$ and $F_{\rm T}$ accordingly. Since $\Theta_{\rm FH} < 0$ the thermal force $F_{\rm T}$ is out of phase with respect to the relative temperature $T_{\rm R}$.

that is formed between the fiber's tip and the mechanical mirror (note that the length of the SC $\simeq 40 \,\mu\text{m}$ is much shorter than the coherence length $\lambda_{\rm L}^2/\Delta\lambda$, where $\Delta\lambda = 0.2 \,\text{nm}$ is the filtering bandwidth of the FBG). Note that the finesse of the SC is by far sufficiently low to allow disregarding retardation in the response of the SC to mechanical displacement.

The effect of the bolometric coupling on the dynamics of the optomechanical cavity has been extensively studied before for the case where light with a constant intensity is externally injected [4, 9–17]. Most results of this analysis are applicable for the SEO measurements shown in Fig. 3, for which the ORC frequency $\omega_{\rm R}$ and the mechanical frequency $\omega_{\rm m}$ are incommensurable. On the other hand, when the detuning between the ORC and mechanical frequencies is sufficiently small, the motioninduced modulation may have a significant effect on the intra-cavity optical intensity $L_{\rm H}$, as is demonstrated by the MML measurements shown in Fig. 4.

In the limit of small mechanical displacement the main effect of the optomechanical coupling originates from two terms of $H_{\rm m} = L_{\rm H}A_{\rm H}$ both oscillating at the mechanical frequency, the first one is due to motion-induced oscillation of the absorption $A_{\rm H}$, and the second one is due to motion-induced modulation in the optical intensity $L_{\rm H}(t)$ [see Eq. (2)]. The effect of both terms can be taken into account by replacing the mechanical angular frequency $\omega_{\rm m}$ and mechanical damping rate $\gamma_{\rm m}$ by effective values given by

$$\omega_{\rm m,eff} = \omega_{\rm m} + \frac{\kappa_{\rm m}}{\omega_{\rm m}} \left(\gamma_{\rm H0} + \gamma_{\rm H1} \right) , \qquad (4)$$

$$\gamma_{\rm m,eff} = \gamma_{\rm m} + \gamma_{\rm H0} + \gamma_{\rm H1} . \tag{5}$$

In both regions of SEO and MML the damping rate $\gamma_{m,eff}$ becomes negative, and consequently the system becomes unstable. Nonlinear corrections to Eqs. (4) and (5) can be evaluated by taking into account both the parametric term proportional to $\Theta_{\rm PH}$ and the second order absorption term proportional to k_{A2} [42]. Note, however, that these nonlinear terms are not needed for determining the threshold of both SEO and MML. The term $\gamma_{\rm H0} = k_{\rm A1} \Theta_{\rm FH} L_0 A_{\rm H0} / (2m_{\rm m} \omega_{\rm m}^2)$ represents the contribution of the average value of the optical intensity L_0 [42], whereas the contribution of the optical intensity $L_{\rm H}$ component oscillating at the mechanical frequency is represented by the term $\gamma_{\rm H1}$. When the detuning between the ORC and mechanical frequencies is negligibly small $\gamma_{\rm H1}$ is found to be given by $\gamma_{\rm H1} = -(2\omega_{\rm m}/T_{\rm N})\gamma_{\rm H0}$ [see Eq. (2)], whereas the term $\gamma_{\rm H1}$ can be disregarded when $\omega_{\rm R}$ and $\omega_{\rm m}$ are incommensurate.

The dominant contribution to the effective noise parameter $T_{\rm N}$ originates from quantum noise of the OA, which has a noise figure $\alpha_{\rm NF}$ given by $\alpha_{\rm NF}$ = $2n_{\rm PI}(G_{\rm OA}-1)/G_{\rm OA}=2.5$, where $G_{\rm OA}=1600$ is the small signal gain and $n_{\rm PI} = 1.25$ is the population inversion parameter [34]. When thermal occupation of the optical modes is negligibly small the effective noise parameter $T_{\rm N}$ is given by $T_{\rm N} \simeq (\gamma_{\rm OM} \alpha_{\rm NF} G_{\rm OA}) / (4 \langle n_{\rm p} \rangle),$ where $\gamma_{\rm OM} \simeq 0.1 \times \omega_{\rm R}$ is a typical mode damping rate (dominated by both insertion loss of the EOAM and radiation loss of the SC), and where $\langle n_{\rm p} \rangle \simeq 2 \times 10^6$ is the averaged photon number per mode for the measurements presented in Fig. 4, and thus for this case $|\gamma_{\rm H1}/\gamma_{\rm H0}| =$ $2\omega_{\rm m}/T_{\rm N} \simeq 4 \times 10^4$ (note that $\langle n_{\rm p} \rangle$ is related to the power P_{OA} delivered by the OA by $P_{\text{OA}} = \gamma_{\text{OM}} \hbar \omega_{\text{p}} N_{\text{R}} \langle n_{\text{p}} \rangle$, where $N_{\rm R} = L_{\rm R} \Delta \lambda / \lambda_{\rm L}^2$. The fact that $|\gamma_{\rm H1}/\gamma_{\rm H0}| \gg 1$ is demonstrated by the experimental observations that MML occurs in almost the entire region of blue detuning (see Fig. 4), whereas for the same optical gain SEO occurs only in a partial region of red detuning (see Fig. 3).

Note that the sign of the term $\gamma_{\rm H1}$ is opposite to the sign of $\gamma_{\rm H0}$. As is explained in the caption of Fig. 5, this can be attributed to the fact that pulses generated by mode locking hit the mirror when the displacementdependent optical absorption $A_{\rm H}(x_{\rm m})$ obtains its minimum value. Both added damping rates $\gamma_{\rm H0}$ and $\gamma_{\rm H1}$ [see Eq. (5)] depend on the detuning of the SC. When the SC is blue (red) detuned the absorption coefficient k_{A1} in Eq. (5) is negative (positive) [43] (note that it is assumed that positive displacement $x_{\rm m}$ is in the outwards direction and that the cavity length $L_{\rm SC}$ is decreased when the piezoelectric motor voltage V_z is increased). Aluminum has a thermal expansion coefficient higher than both silicon and silicon-nitride, and consequently it is expected that $\Theta_{\rm FH} < 0$ and $\Theta_{\rm PH} < 0$. Thus, for the device under study here it is expected that $\gamma_{\rm H0} < 0$ when the SC is red detuned [see Eq. (5)]. This behavior is demonstrated by the SEO shown in Fig. 3. On the other hand, $\gamma_{\rm H1} < 0$ when the SC is blue detuned. This is consistent with the experimental observation that MML occurs mainly with blue detuning (see Fig. 4).

In summary, we find that mode locking can be obtained by integrating gain medium into an optomechanical cavity. The threshold optical power for MML is found to be significantly lower than the corresponding value for SEO. Future study will explore applications of MML for sensing. For example, Braginsky has proposed a device called a speed meter, which allows monitoring a classical force acting on a mechanical resonator with sensitivity that can exceed the so-called standard quantum limit [27, 44]. In the steady state of MML the pulses hit the mirror when its velocity nearly vanishes, and thus the off reflected pulses mainly carry information about the velocity (rather than the position) of the mirror, therefore a sensor based on MML may serve as a sensitive speed meter.

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