The mean field approximation and disentanglement

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The mean field approximation becomes applicable when entanglement is sufficiently weak. We explore a nonlinear term that can be added to the Schrödinger equation without violating unitarity of the time evolution. We find that the added term suppresses entanglement, without affecting the evolution of any product state. The dynamics generated by the modified Schrödinger equation is explored for the case of a two-spin 1/2 system. We find that for this example the added term strongly affects the dynamics when the Hartmann Hahn matching condition is nearly satisfied.

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I. INTRODUCTION

Consider a system made of two subsystems. $A = A^{\dagger}$ ($B = B^{\dagger}$) be a measurable of the first (second) subsystem. In the mean field approximation [1-3] it is assumed that $\langle AB \rangle = \langle A \rangle \langle B \rangle$ (angle brackets denote an expectation value). This approximation is valid when entanglement is sufficiently small, and it becomes exact for any product state. Let $Q(|\psi\rangle)$ be the level of entanglement of a given ket state vector $|\psi\rangle$. There are several different ways to quantify entanglement [4], however, none is linear in $|\psi\rangle$, i.e. generally $Q(|\psi_1\rangle + |\psi_2\rangle) \neq Q(|\psi_1\rangle) + Q(|\psi_2\rangle)$. On the other hand, the Schrödinger equation is linear in $|\psi\rangle$. Moreover, the Gorini Kossakowski Sudarshan Lindblad equation (GKSL) master equation [5, 6] is linear in the density operator ρ . Hence, a process which gives rise to disentanglement only, without affecting the dynamics of product states, cannot be properly described with time evolution that is generated either by Schrödinger or GKSL equations

Here we consider a modified Schrödinger equation, which includes a nonlinear term that suppresses entanglement. The proposed equation can be constructed for any physical system whose Hilbert space has finite dimensionality.

Previously proposed nonlinear terms that can be added to the Schrödinger equation are reviewed in [7]. Weinberg has considered a class of nonlinear Schrödinger equations, for which combining subsystems is possible [8, 9]. The time evolution of the probability density generated by a nonlinear Schrödinger equation has been studied using the Fokker-Planck equation in [10]. Gauge inveriance in nonlinear Schrödinger equations has been explored in [11]. In most previous proposals, the purpose of the added nonlinear terms is to generate a spontaneous collapse [12–16].

On the hand, here we propose an added nonlinear term that generates disentanglement, i.e. it suppresses entanglement without affecting the dynamics of product states. We explore the effect of the added disentanglement nonlinear term for the case of a two-spin 1/2 system. With externally applied driving the two-spin 1/2 system can become unstable [17, 18] when the Hartmann

Hahn matching condition [19, 20] is nearly satisfied. We find that in the same region the added nonlinear disentangling term has a relatively large effect on the dynamics.

II. THE SCHMIDT DECOMPOSITION

Consider a system composed of two subsystems labeled as '1' and '2', respectively. The dimensionality of the Hilbert spaces of both subsystems, which is denoted by N_1 and N_2 , respectively, is assumed to be finite. The system is in a normalized pure state vector $|\psi\rangle$ given by

$$|\psi\rangle = \mathcal{K}_1 C \otimes \mathcal{K}_2^{\mathrm{T}} , \qquad (1)$$

where C is a $N_1 \times N_2$ matrix having entries C_{k_1,k_2} , matrix transposition is denoted by T, the raw vectors \mathcal{K}_1 and \mathcal{K}_1 are given by

$$\mathcal{K}_1 = (|k_1\rangle_1, |k_2\rangle_1, \cdots, |k_{N_1}\rangle_1) , \qquad (2)$$

$$\mathcal{K}_2 = (|k_1\rangle_2, |k_2\rangle_2, \cdots, |k_{N_2}\rangle_2) , \qquad (3)$$

and $\{|k_1\rangle_1\}$ ($\{|k_2\rangle_2\}$) is an orthonormal basis spanning the Hilbert space of subsystem '1' ('2').

The purity P_1 (P_2) is defined by $P_1 = \text{Tr } \rho_1^2$ ($P_2 = \text{Tr } \rho_2^2$), where $\rho_1 = \text{Tr}_2 \rho$ ($\rho_2 = \text{Tr}_1 \rho$) is the reduced density operator of the first (second) subsystems. By employing the Schmidt decomposition one finds that $P_1 = P_2 \equiv P$, and that the level of entanglement Q, which is defined by Q = 1 - P, is given by [see Eq. (8.107)) in [21]

$$Q = 2 \sum_{k_1' < k_1'' k_2' < k_2''} \left| \left\langle \Psi_{k_1', k_1'', k_2', k_2''} \right| \psi \right\rangle \right|^2 , \qquad (4)$$

where the state $\langle \Psi_{k'_1,k''_1,k'_2,k''_2}|$, which depends on on the matrix C corresponding to a given state $|\psi\rangle$, is given by (note that $\langle \Psi_{k'_1,k''_1,k'_2,k''_2}|$ is not normalized)

$$\left\langle \Psi_{k_{1}^{\prime},k_{1}^{\prime\prime},k_{2}^{\prime},k_{2}^{\prime\prime}}\right| = C_{k_{1}^{\prime\prime},k_{2}^{\prime\prime}} \left\langle k_{1}^{\prime},k_{2}^{\prime}\right| - C_{k_{1}^{\prime\prime},k_{2}^{\prime}} \left\langle k_{1}^{\prime},k_{2}^{\prime\prime}\right| \; . \quad (5)$$

Note that Q = 0 for a product state, and that Q is time independent when the subsystems are decoupled (i.e. their mutual interaction vanishes).

As an example, consider a two spin 1/2 system (i.e. $N_1 = N_2 = 2$) in a pure state $|\psi\rangle$ given by

$$|\psi\rangle = a|--\rangle + b|-+\rangle + c|+-\rangle + d|++\rangle$$
 . (6)

For this case Eq. (4) yields $Q = 2 |\langle \Psi | \psi \rangle|^2$, where [for this case the sum in Eq. (4) contains a single term with $k'_1 = -, k''_1 = +, k'_2 = -$ and $k''_2 = +$]

$$\langle \Psi | = d \langle --| - c \langle -+| , \qquad (7)$$

hence $Q = 2 |ad - bc|^2$. The following holds $Q \leq 1/2$ provided that $|\psi\rangle$ is normalized [4].

III. DISENTANGLEMENT

As will be shown below, entanglement can be suppressed by adding appropriate nonlinear terms to the Schrödinger equation. Consider a modified Schrödinger equation for the ket vector $|\psi\rangle$ having the form

$$\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \left(-i\hbar^{-1}\mathcal{H} + \gamma_{\mathrm{D}}M_{\mathrm{D}}\right)|\psi\rangle , \qquad (8)$$

where \hbar is the Planck's constant, \mathcal{H} is the Hamiltonian, the rate γ_D is positive, the operator M_D is given by

$$M_{\rm D} = -\sqrt{\frac{\langle \Psi | \Psi \rangle}{1 - \langle \mathcal{P} \rangle}} \left(\mathcal{P} - \langle \mathcal{P} \rangle \right) , \qquad (9)$$

the projection operator \mathcal{P} is given by

$$\mathcal{P} = \frac{|\Psi\rangle \langle \Psi|}{\langle \Psi | \Psi\rangle} \,, \tag{10}$$

the expectation value $\langle \mathcal{P} \rangle$ is given by

$$\langle \mathcal{P} \rangle = \frac{\langle \psi | \mathcal{P} | \psi \rangle}{\langle \psi | \psi \rangle} = \frac{\left| \langle \Psi | \psi \rangle \right|^2}{\langle \Psi | \Psi \rangle \langle \psi | \psi \rangle} , \qquad (11)$$

where $|\Psi\rangle$ is a given ket vector.

Note that $M_{\rm D} | \psi \rangle = 0$ provided that $\langle \Psi | \psi \rangle = 0$, thus the added term has no effect when $| \psi \rangle$ is orthogonal to $| \Psi \rangle$. On the other hand, any product state $| \psi \rangle$ is orthogonal to all the vectors $| \Psi_{k'_1, k''_1, k'_2, k''_2} \rangle$ given by Eq. (5). Thus, any added term having the form $\gamma_{\rm D} M_{\rm D}$, which is constructed based on a vector $| \Psi \rangle$ that is one of the vectors $| \Psi_{k'_1, k''_1, k'_2, k''_2} \rangle$ given by Eq. (5), has no effect when $| \psi \rangle$ represents a product state.

Using Eqs. (9) and (10) one finds that (note that $\mathcal{P}^2 = \mathcal{P}$)

$$\langle \psi | M_{\rm D} | \psi \rangle = 0 , \qquad (12)$$

$$\langle \Psi | M_{\rm D} | \psi \rangle = -\sqrt{\langle \Psi | \Psi \rangle (1 - \langle \mathcal{P} \rangle)} \langle \Psi | \psi \rangle , \qquad (13)$$

$$\langle \psi | M_{\rm D}^2 | \psi \rangle = |\langle \Psi | \psi \rangle|^2 . \tag{14}$$

The relation (12) implies that $M_{\rm D} | \psi \rangle$ is orthogonal to $| \psi \rangle$, and thus the unitarity condition, which reads [see

Eq. (8)] $0 = (d/dt) \langle \psi | \psi \rangle = i\hbar^{-1} \langle \psi | \mathcal{H}^{\dagger} - \mathcal{H} | \psi \rangle + 2\gamma_{\rm D} \operatorname{Re} \langle \psi | M_{\rm D} | \psi \rangle$, is satisfied provided that the Hamiltonian is Hermitian, i.e. $\mathcal{H}^{\dagger} = \mathcal{H}$ (henceforth it is assumed that \mathcal{H} is Hermitian).

The modified Schrödinger equation (8) yields a modified Heisenberg equation given by

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle \psi | O | \psi \rangle = \frac{\langle \psi | [O, \mathcal{H}] | \psi \rangle}{i\hbar} + \gamma_{\mathrm{D}} \langle \psi | \{O, M_{\mathrm{D}}\} | \psi \rangle , \qquad (15)$$

where $O = O^{\dagger}$ is a given observable, that does not explicitly depend on time, and where $\{O, M_{\rm D}\} = OM_{\rm D} + M_{\rm D}O$. For the case where $O = \mathcal{P}$ Eq. (15) yields [see Eq. (13)]

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\left| \langle \Psi | \psi \rangle \right|^{2}}{\left\langle \Psi | \Psi \rangle} = \frac{\left\langle \psi \right| \left[\mathcal{P}, \mathcal{H} \right] | \psi \rangle}{i\hbar}
- 2\gamma_{\mathrm{D}} \sqrt{\left\langle \Psi | \Psi \rangle \left(1 - \left\langle \mathcal{P} \right\rangle \right)} \frac{\left| \langle \Psi | \psi \rangle \right|^{2}}{\left\langle \Psi | \Psi \rangle} .$$
(16)

An upper bound can be derived for the first term on the right hand side of Eq. (16) using the uncertainty principle $|\langle [\mathcal{P}, \mathcal{H}] \rangle|^2 \leq 4 \left\langle (\mathcal{P} - \langle \mathcal{P} \rangle)^2 \right\rangle \left\langle (\mathcal{H} - \langle \mathcal{H} \rangle)^2 \right\rangle \leq \left\langle (\mathcal{H} - \langle \mathcal{H} \rangle)^2 \right\rangle$ [note that $\left\langle (\mathcal{P} - \langle \mathcal{P} \rangle)^2 \right\rangle = \left\langle \mathcal{P}^2 \right\rangle - \left\langle \mathcal{P} \right\rangle^2$, $\mathcal{P}^2 = \mathcal{P}, \ 0 \leq \langle \mathcal{P} \rangle \leq 1$ and $0 \leq 4x (1-x) \leq 1$ for $0 \leq x \leq 1$]. The second term on the right hand side of Eq. (16) represents a rotation of the ket vector $|\psi\rangle$ away from the ket vector $|\Psi\rangle$. This rotation gives rise to disentanglement when $|\Psi\rangle$ is chosen to be one of the vectors $|\Psi_{k_1',k_1'',k_2',k_2''}\rangle$ given by Eq. (5).

IV. TWO-SPIN SYSTEM

As an example, consider two two-level systems (TLS) having a mutual coupling that is characterized by a coupling coefficient g. The first TLS, which is labelled as 'a', has a relatively low angular frequency $\omega_{\rm a}$ in comparison with the angular frequency $\omega_{\rm b}$ of the second TLS, which is labelled as 'b', and which is externally driven. The Hamiltonian \mathcal{H} of the closed system is given by

$$\mathcal{H} = \omega_{a} S_{az} + \omega_{b} S_{bz} + \frac{\omega_{1} (S_{b+} + S_{b-})}{2} + g \hbar^{-1} (S_{a+} + S_{a-}) S_{bz} ,$$
(17)

where the driving amplitude and angular frequency are denoted by ω_1 and $\omega_p = \omega_b + \Delta$, respectively (Δ is the driving detuning), the operators $S_{a\pm}$ are given by $S_{a\pm} = S_{ax} \pm iS_{ay}$, and the rotated operators $S_{b\pm}$ are given by $S_{b\pm} = (S_{bx} \pm iS_{by}) e^{\pm i\omega_p t}$. In the basis $\{|--\rangle, |-+\rangle, |+-\rangle, |++\rangle$ the matrix representation of the Hamiltonian is given by $\hbar\Omega$, where the matrix Ω is

given by

$$\Omega = \begin{pmatrix}
\frac{-\omega_{a} - \omega_{b}}{2} & \frac{\omega_{1}}{2} e^{i\omega_{p}t} & -\frac{g}{2} & 0 \\
\frac{\omega_{1}}{2} e^{-i\omega_{p}t} & \frac{-\omega_{a} + \omega_{b}}{2} & 0 & \frac{g}{2} \\
-\frac{g}{2} & 0 & \frac{\omega_{a} - \omega_{b}}{2} & \frac{\omega_{1}}{2} e^{i\omega_{p}t} \\
0 & \frac{g}{2} & \frac{\omega_{1}}{2} e^{-i\omega_{p}t} & \frac{\omega_{a} + \omega_{b}}{2}
\end{pmatrix} . (18)$$

The unitary transformations $U_1 = 1_a \otimes u_{b1}$ and $U_2 = 1_a \otimes u_{b2}$ are successively employed below, where 1_a is the identity operator of TLS a. For the first one, which transforms TLS b to a frame rotated at the angular driving frequency ω_p , the matrix representation of u_{b1} is given by

$$u_{\rm bl} \doteq \begin{pmatrix} e^{\frac{i\omega_{\rm p}t}{2}} & 0\\ 0 & e^{-\frac{i\omega_{\rm p}t}{2}} \end{pmatrix} , \qquad (19)$$

and the corresponding transformed matrix Ω' is given by [see Eq. (18) and Eq. (6.329) in [21]]

$$\Omega' = \begin{pmatrix} \frac{-\omega_{a} + \Delta}{2} & \frac{\omega_{1}}{2} & -\frac{g}{2} & 0\\ \frac{\omega_{1}}{2} & \frac{-\omega_{a} - \Delta}{2} & 0 & \frac{g}{2}\\ -\frac{g}{2} & 0 & \frac{\omega_{a} + \Delta}{2} & \frac{\omega_{1}}{2}\\ 0 & \frac{g}{2} & \frac{\omega_{1}}{2} & \frac{\omega_{a} - \Delta}{2} \end{pmatrix} . \tag{20}$$

For the second transformation the matrix representation of u_{b2} is given by

$$u_{\rm b2} \doteq \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} , \qquad (21)$$

where

$$\tan \theta = -\frac{\omega_1}{\Lambda} \,\,\,\,(22)$$

and the corresponding transformed matrix Ω'' is given by [see Eq. (20)]

$$\Omega'' = \begin{pmatrix}
\frac{-\omega_{\rm a} + \omega_{\rm R}}{2} & 0 & -\frac{g\Delta}{2\omega_{\rm R}} & \frac{g\omega_{\rm 1}}{2\omega_{\rm R}} \\
0 & -\frac{\omega_{\rm a} - \omega_{\rm R}}{2} & \frac{g\omega_{\rm 1}}{2\omega_{\rm R}} & \frac{g\Delta}{2\omega_{\rm R}} \\
-\frac{g\Delta}{2\omega_{\rm R}} & \frac{g\omega_{\rm 1}}{2\omega_{\rm R}} & \frac{\omega_{\rm a} + \omega_{\rm R}}{2} & 0 \\
\frac{g\omega_{\rm 1}}{2\omega_{\rm R}} & \frac{g\Delta}{2\omega_{\rm R}} & 0 & \frac{\omega_{\rm a} - \omega_{\rm R}}{2}
\end{pmatrix} , (23)$$

where $\omega_{\rm R}$, which is given by

$$\omega_{\rm R} = \sqrt{\omega_1^2 + \Delta^2} \,, \tag{24}$$

is the Rabi angular frequency.

The matrix Ω'' in the limit where the TLSs are decoupled is denoted by Ω''_0 , i.e. $\Omega''_0 = \lim_{g\to 0} \Omega''$. The first and forth energy eigenvalues of Ω''_0 become degenerate when the Hartmann Hahn matching condition $\omega_a = \omega_R$ is satisfied. Consequently, the effect of the coupling becomes relatively strong when $\omega_a \simeq \omega_R$. In this region the problem can be simplified by employing a truncation approximation into the subspace spanned by the transformed states $|--\rangle$ and $|++\rangle$ [first and forth vectors of the basis that is used for constructing the matrix Ω''

given by Eq. (23)]. In this approximation the 4×4 matrix Ω'' is replaced by the 2×2 truncated matrix $\Omega''_{\rm T}$, which is given by

$$\Omega_{\rm T}^{"} = \begin{pmatrix} \frac{\omega_{\rm R} - \omega_{\rm a}}{2} & \frac{g\omega_{\rm 1}}{2\omega_{\rm R}} \\ \frac{g\omega_{\rm 1}}{2\omega_{\rm B}} & -\frac{\omega_{\rm R} - \omega_{\rm a}}{2} \end{pmatrix} = \boldsymbol{\omega} \cdot \boldsymbol{\sigma} , \qquad (25)$$

where the effective magnetic field vector $\boldsymbol{\omega}$ is given by

$$\boldsymbol{\omega} = \left(\frac{g\omega_1}{2\omega_R}, 0, \frac{\omega_R - \omega_a}{2}\right) \equiv (\omega_x, 0, \omega_z) , \qquad (26)$$

and the components of the Pauli matrix vector $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(27)$$

The notation given by Eq. (6) is employed below for the transformed state $|\psi''\rangle$. It is henceforth assumed that $|\psi''\rangle$ is normalized, i.e. $|a|^2+|d|^2=1$ (note that b=c=0 for the truncation approximation). The polarization vector \mathbf{P} is given by $\mathbf{P}=(\langle\psi|\sigma_x|\psi\rangle,\langle\psi|\sigma_y|\psi\rangle,\langle\psi|\sigma_z|\psi\rangle)$, where $P_x=d^*a+a^*d$, $P_y=i(d^*a-a^*d)$ and $P_z=|a|^2-|d|^2$. Note that $\mathbf{P}\cdot\mathbf{P}=1$ provided that $|\psi''\rangle$ is normalized.

In the truncation approximation $M_{\rm D}$ is replaced by the 2×2 matrix $M_{\rm DT}$, which is given by [see Eqs. (7) and (9)]

$$M_{\rm DT} = \begin{pmatrix} -|d|^2 & 0\\ 0 & |a|^2 \end{pmatrix} = \frac{|a|^2 - |d|^2}{2} - \frac{\sigma_z}{2} .$$
 (28)

The following holds $\langle \psi'' | M_{\rm DT}^2 | \psi'' \rangle = |ad|^2 = Q/2$, where Q is the level of entanglement [recall that $|a|^2 + |d|^2 = 1$, and compare with Eq. (14)].

With the help of the identity

$$(\boldsymbol{\sigma} \cdot \mathbf{a}) (\boldsymbol{\sigma} \cdot \mathbf{b}) = \mathbf{a} \cdot \mathbf{b} + i \boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b}) , \qquad (29)$$

where **a** and **b** are given vectors, one finds that [recall the vector identity $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$, and see Eqs. (15), (25) and (28)]

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = 2\boldsymbol{\omega} \times \mathbf{P} + \gamma_{\mathrm{D}} \mathbf{V}_{\mathrm{D}} , \qquad (30)$$

where the vector \mathbf{V}_{D} is given by [note that $\{\sigma_x, \sigma_z\} = \{\sigma_y, \sigma_z\} = 0$ and $\{\sigma_z, \sigma_z\} = 2$, see Eq. (29)]

$$\mathbf{V}_{\mathrm{D}} = P_z \mathbf{P} - \hat{\mathbf{z}} \,, \tag{31}$$

where $\hat{\mathbf{z}}$ is a unit vector in the z direction. The following holds $\mathbf{P} \cdot \mathbf{V}_{\mathrm{D}} = 0$ [compare with Eq. (12)] and $\mathbf{V}_{\mathrm{D}} \cdot \mathbf{V}_{\mathrm{D}} = 1 - P_{z}^{2} = 4 |ad|^{2} = 2Q$ [recall that $\mathbf{P} \cdot \mathbf{P} = 1$ and $P_{z} = |a|^{2} - |d|^{2}$, and compare with Eq. (14)]. For a normalized \mathbf{P} the following holds $\mathbf{V}_{\mathrm{D}} = (\hat{\mathbf{z}} \times \mathbf{P}) \times \mathbf{P}$, hence Eq. (30) can be rewritten as

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = (2\boldsymbol{\omega} + \gamma_{\mathrm{D}}\hat{\mathbf{z}} \times \mathbf{P}) \times \mathbf{P} . \tag{32}$$

Note that when $\boldsymbol{\omega} \parallel \hat{\mathbf{z}}$ the equation of motion (32) is similar to the Landau–Lifshitz equation for the time evolution of the magnetization vector of a ferromagnet. However, the condition $\boldsymbol{\omega} \parallel \hat{\mathbf{z}}$, which is satisfied only when the driving amplitude ω_1 vanishes [see Eq. (26)], is inconsistent with the assumption that the Hartmann Hahn matching condition is nearly satisfied.

Let $\mathbf{P}_0 = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ be a fixed point of Eq. (32) (i.e. $d\mathbf{P}/dt = 0$ at \mathbf{P}_0). Using Eq. (32) one finds that the angle θ for the fixed point \mathbf{P}_0 can be found by solving

$$\frac{\gamma_{\rm D}^2}{4(\omega_x^2 + \omega_z^2)} = \frac{\sin(\theta_{\rm H} - \theta)\sin(\theta_{\rm H} + \theta)}{\sin^2\theta\cos^2\theta} , \quad (33)$$

where

$$\tan \theta_{\rm H} = \frac{\omega_x}{\omega_z} \,. \tag{34}$$

According to Eq. (33), in the absence of disentanglement, i.e. when $\gamma_{\rm D}=0$, the vector \mathbf{P}_0 is parallel to the vector $\boldsymbol{\omega}$ (26), whereas in the opposite limit of strong disentanglement, i.e. when $\gamma_{\rm D}^2\gg\omega_x^2+\omega_z^2$, the vector \mathbf{P}_0 becomes nearly parallel to $\hat{\mathbf{z}}$ (i.e. the state represented by the fixed point \mathbf{P}_0 nearly becomes a product state).

V. SUMMARY

In summary, the modified Schrödinger equation suppresses entanglement without violating unitarity. Future study will be devoted to the rich nonlinear dynamics that is generated by the added disentanglement term. Some

outstanding questions, which were left outside the scope of the current manuscript, are briefly mentioned below. How a given system should be divided into two (or perhaps more) subsystems? In traditional quantum mechanics such a division is generally not unique. How to determine the values of the disentanglement rates? The hypothesis that these rates remain finite even when the subsystems become decoupled is likely to be inconsistent with the principle of causality [22, 23] (recall that in traditional quantum mechanics the level of entanglement Q becomes time independent when the subsystems are decoupled). Recently, it was shown that when a condition, called 'convex quasilinearity' is satisfied by a given nonlinear Schrödinger equation, violation of the causality principle becomes impossible [7]. Upper bound imposed upon the disentanglement rates can be derived from experimental observations of quantum entanglement.

The existence of quantum entanglement has been conclusively demonstrated in many different physical systems. On the other hand, entanglement can be held responsible for a fundamental self-inconsistency related to the quantum to classical transition [24–26], which was first introduced by Schrödinger [27] (this self-inconsistency is commonly known as the problem of quantum measurement). Exploring possible mechanisms of disentanglement may help resolving this long-standing self-inconsistency.

VI. ACKNOWLEDGMENTS

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