## Partial disentanglement in a multipartite system

Eyal Buks\*

Andrew and Erna Viterbi Department of Electrical Engineering, Technion, Haifa 32000, Israel

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We explore a nonlinear extension to quantum theory giving rise to deterministic partial disentanglement between pairs of subsystems. The extension is based on a modified Schrödinger equation having an added nonlinear term. To avoid conflicts with the principles of causality and separability, it is postulated that disentanglement is active only during the time when particles interact. A butterfly-like effect is found near highly entangled multipartite vector states.

Introduction - The problem of quantum measurement [1] arguably originates from a self-inconsistency in quantum theory [2–4]. In an attempt to resolve this longstanding problem, several nonlinear extensions to quantum theory [5] have been proposed [6–12], and processes giving rise to spontaneous collapse have been explored [13–20]. For some cases, however, nonlinear quantum dynamics may give rise to conflicts with well-established physical principles, such as causality [21–26] and separability [22, 27, 28]. In addition, some predictions of standard quantum mechanics, which have been experimentally confirmed to very high accuracy, are inconsistent with some of the proposed extensions.

A nonlinear mechanism giving rise to suppression of entanglement (i.e. disentanglement) has been recently proposed [29]. This mechanism of disentanglement, which makes the collapse postulate redundant, is introduced by adding a nonlinear term to the Schrödinger equation. The proposed modified Schrödinger equation can be constructed for any physical system whose Hilbert space has finite dimensionality, and it does not violate norm conservation of the time evolution. The nonlinear term added to the Schrödinger equation has no effect on product (i.e. disentangled) states.

The derivation of the added term for the case of a system composed of two subsystems is given in Ref. [29]. A system belonging to this class is henceforth referred to as a bipartite. The main purpose of the current paper is to propose a generalization, applicable for the case where the system under study is divided into more than two subsystems (the multipartite case). The derivation of the nonlinear term added to the Schrödinger equation for the multipartite case, which is discussed below, is related to the quantification problem of subsystems' quantum entanglement [30–40].

**Partial disentanglement** - Consider a modified Schrödinger equation for the ket vector  $|\psi\rangle$  having the form

$$\frac{\mathrm{d}}{\mathrm{d}t} \left| \psi \right\rangle = \left[ -i\hbar^{-1} \mathcal{H} - \gamma \left( \mathcal{Q} - \left\langle \mathcal{Q} \right\rangle \right) \right] \left| \psi \right\rangle , \qquad (1)$$

where  $\hbar$  is the Planck's constant,  $\mathcal{H} = \mathcal{H}^{\dagger}$  is the Hamiltonian, the rate  $\gamma$  is positive, the operator  $\mathcal{Q}$  is allowed

to depend on  $|\psi\rangle$ , and  $\langle Q \rangle \equiv \langle \psi | Q | \psi \rangle$ . Note that the norm conservation condition  $0 = (d/dt) \langle \psi | \psi \rangle$  is satisfied by the modified Schrödinger equation (1), provided that  $|\psi\rangle$  is normalized, i.e.  $\langle \psi | \psi \rangle = 1$ . As is shown in appendix A, the added nonlinear term proportional to  $\gamma$  in Eq. (1) suppresses the expectation value  $\langle Q \rangle$  (provided that  $\gamma$  is positive). In particular, this term can be constructed to suppress entanglement, i.e. to give rise to disentanglement. The case of a bipartite system was discussed in [29]. Here we consider the more general case of a multipartite system, and derive a modified Schrödinger equation having the form of Eq. (1), which can give rise to partial disentanglement between any pair of subsystems.

Consider a multipartite system composed of three subsystems labeled as '1', '2' and '3', respectively. The Hilbert space of the system  $H = H_3 \otimes H_2 \otimes H_1$  is a tensor product of subsystem Hilbert spaces  $H_1$ ,  $H_2$  and  $H_3$ . The dimensionality of the Hilbert space  $H_n$  of subsystem n, which is denoted by  $d_n$ , where  $n \in \{1, 2, 3\}$ , is assumed to be finite. The system is assumed to be in a pure state, which is denoted by  $|\psi\rangle$ .

For any given observable  $A_1 = A_1^{\dagger}$  of subsystem 1, and a given observable  $A_2 = A_2^{\dagger}$  of subsystem 2, the operator  $C_{12}(A_1, A_2)$  is defined by

$$\mathcal{C}_{12}(A_1, A_2) = A_2 A_1 |\psi\rangle \langle\psi| - A_1 |\psi\rangle \langle\psi| A_2. \quad (2)$$

Subsystems 1 and 2 are said to be disentangled if  $\langle C_{12}(A_1, A_2) \rangle = 0$  for any single particle observables  $A_1$  and  $A_2$ . A general single particle observable of subsystem n, where  $n \in \{1, 2\}$ , can be expanded using the set of generalized Gell-Mann matrices  $\left\{\lambda_1^{(n)}, \lambda_2^{(n)}, \cdots, \lambda_{d_n^2-1}^{(n)}\right\}$ , which spans the SU $(d_n)$  Lie algebra. The entanglement between subsystems 1 and 2 can be quantified by the nonnegative variable  $\tau_{12}$ , which is given by  $\tau_{12} = \langle Q_{12} \rangle$ , where the operator  $Q_{12}$  is given by [compare to Eq. (31) of Ref. [30]]

$$\mathcal{Q}_{12} = \eta_{12} \sum_{a_1=1}^{d_1^2 - 1} \sum_{a_2=1}^{d_2^2 - 1} \mathcal{C}_{12} \left( \lambda_{a_1}^{(1)}, \lambda_{a_2}^{(2)} \right) |\psi\rangle \langle \psi| \, \mathcal{C}_{12} \left( \lambda_{a_1}^{(1)}, \lambda_{a_2}^{(2)} \right)$$
(3)

and where  $\eta_{12}$  is a positive constant.

In a similar way, the entanglement between subsystems 2 and 3, which is denoted by  $\tau_{23}$ , and the entanglement

<sup>\*</sup>Electronic address: eyal@ee.technion.ac.il

between subsystems 3 and 1, which is denoted by  $\tau_{31}$ , can be defined. The completeness relation that is satisfied by the generalized Gell-Mann matrices [see Eq. (8.166) of [41]] can be used to show that  $\tau_{23}$ ,  $\tau_{31}$  and  $\tau_{12}$  are all invariant under any single subsystem unitary transformation [30]. Deterministic disentanglement between subsystems n' and n'' can be generated by the modified Schrödinger equation (1), provided that the operator Qin Eq. (1) is replaced by the operator  $Q_{n',n''}$ .

Three spin 1/2 system - Partial disentanglement is explored below for the relatively simple case of a system composed of three spin 1/2particles. For this case  $d_1 = d_2 = d_3 = 2$ ,  $d_n^2 - 1 = 3$ , and the set of generalized Gell-Mann matrices for the *n*'th spin is the set of Pauli matrices  $\{2\hbar^{-1}\mathbf{S}_n \cdot \hat{\mathbf{x}}, 2\hbar^{-1}\mathbf{S}_n \cdot \hat{\mathbf{y}}, 2\hbar^{-1}\mathbf{S}_n \cdot \hat{\mathbf{z}}\} \doteq \{\lambda_1^{(n)}, \lambda_2^{(n)}, \lambda_3^{(n)}\},$ where  $\mathbf{S}_n = (S_{nx}, S_{ny}, S_{nz})$  is the angular momentum vector operator of the *n*'th spin (the symbol  $\doteq$  stands for matrix representation), and  $n \in \{1, 2, 3\}$ . Consider a pure normalized state  $|\psi\rangle$  given by

$$\begin{aligned} |\psi\rangle &= q_{000} |000\rangle + q_{001} |001\rangle + q_{010} |010\rangle + q_{011} |011\rangle \\ &+ q_{100} |100\rangle + q_{101} |101\rangle + q_{110} |110\rangle + q_{111} |111\rangle . \end{aligned}$$
(4)

The ket vector  $|\sigma_3\sigma_2\sigma_1\rangle$ , where  $\sigma_n \in \{0,1\}$  and  $n \in \{1,2,3\}$ , represents an eigenvector of the matrix  $(1 - \lambda_3^{(n)})/2 = \text{diag}(0,1)$  with the eigenvalue  $\sigma_n$ , for n = 1, n = 2, and n = 3. The state  $|\psi\rangle$  is characterized by the single spin Bloch vectors  $\mathbf{k}_n = (\langle \lambda_1^{(n)} \rangle, \langle \lambda_2^{(n)} \rangle, \langle \lambda_3^{(n)} \rangle)$ , for n = 1, n = 2, and n = 3. The length of the n'th Bloch vectors  $\mathbf{k}_n$ , which is denoted by  $k_n = |\mathbf{k}_n|$ , is bounded between zero (fully entangled) and unity (fully disentangled, or fully separable).

For this three spin 1/2 system, the partial entanglements  $\tau_{23}$ ,  $\tau_{31}$  and  $\tau_{12}$  are bounded between zero and unity, provided that  $\eta_{23} = \eta_{31} = \eta_{12} = 1/3$  [30]. Disentanglement between subsystems 1 and 2 is studied below using the modified Schrödinger equation (1), with  $\mathcal{H} = 0$ , and with the operator  $\mathcal{Q}$  taken to be given by  $\mathcal{Q} = \mathcal{Q}_{12}$ [see Eq. (3), and Eq. (8.163) of Ref. [41]]. For this case, the time evolution governed by Eq. (1) gives rise to monotonic decrease of  $\tau_{12}$  in time t.

For the plots shown in Fig. 1 and Fig. 2, the initial state vector, which is denoted by  $|\psi_i\rangle$ , is close to the Greenberger Horne Zeilinger fully entangled state  $|\psi_{\rm GHZ}\rangle$ , which is given by [42]

$$|\psi_{\rm GHZ}\rangle = \frac{|000\rangle - |111\rangle}{\sqrt{2}} . \tag{5}$$

The plots in Fig. 1 exhibit time evolution from initial state  $|\psi_i\rangle$  at time t = 0 (labelled by green star symbols), to final state  $|\psi_f\rangle$  at time  $t = 50/\gamma$  (labelled by red star symbols). For the *n*'th spin, the Bloch vector is shown in Fig. 1(an), the Bloch vector length  $k_n$  as a function of time in Fig. 1(bn), and the corresponding complement subsystem partial entanglement, i.e.  $\tau_{23}$  for  $n = 1, \tau_{31}$ 



FIG. 1: Time evolution for  $|\psi_i\rangle \simeq |\psi_{\rm GHZ}\rangle$ . The initial state, which is given by  $|\psi_i\rangle \sim |\psi_{\rm GHZ}\rangle - 10^{-5}i |\psi_{\rm B,1}(\pi)\rangle - 5.2 \times 10^{-4}i |\psi_{\rm B,2}(\pi)\rangle$ , is represented by green star symbols, and the final state at time  $t = 50/\gamma$  by red star symbols.



FIG. 2: Partial entanglements for  $|\psi_i\rangle \sim |\psi_{\text{GHZ}}\rangle + i\epsilon_1 |\psi_{\text{B},1}(\pi)\rangle + i\epsilon_2 |\psi_{\text{B},2}(\pi)\rangle$ . The values of  $\tau_{23}$ ,  $\tau_{31}$ , and  $\tau_{12}$  at the final time  $t = 50/\gamma$  are shown in (1), (2) and (3), respectively, as a function of  $\epsilon_1$  and  $\epsilon_2$ . The overlaid white  $\times$  symbol represents the initial state  $|\psi_i\rangle$  used for generating the plots in Fig. 1.

for n = 2,  $\tau_{12}$  for n = 3, as a function of time in Fig. 1(cn).

For the example shown in Fig. 1, the initial state  $|\psi_i\rangle$ is given by  $|\psi_i\rangle \sim |\psi_{\text{GHZ}}\rangle + i\epsilon_1 |\psi_{\text{B},1}(\pi)\rangle + i\epsilon_2 |\psi_{\text{B},2}(\pi)\rangle$ , where  $\epsilon_1 = -10^{-5}$  and  $\epsilon_2 = -5.2 \times 10^{-4}$ . The symbol  $\sim$ stands for equality up to normalization, i.e.  $|\psi_i\rangle \sim |\psi'\rangle$ implies that  $|\psi_i\rangle = |\psi'\rangle / ||\psi'\rangle||$ . The states  $|\psi_{\text{B},n}(\theta)\rangle$ , where  $n \in \{1, 2, 3\}$ , and where  $\theta$  is real, are two-spin Bell



FIG. 3: Time evolution for  $|\psi_i\rangle \simeq |\psi_{\rm B,3}(\pi)\rangle$ . The initial state, which is given by  $|\psi_i\rangle \sim |\psi_{\rm B,3}(\pi)\rangle + 9 \times 10^{-5}i |\psi_{\rm B,2}(\pi)\rangle$ , is represented by green star symbols, and the final state at time  $t = 50/\gamma$  by red star symbols.

states given by

$$|\psi_{\mathrm{B},1}\left(\theta\right)\rangle = \frac{|000\rangle + e^{i\theta} |110\rangle}{\sqrt{2}} , \qquad (6)$$

$$|\psi_{\mathrm{B},2}\left(\theta\right)\rangle = \frac{|000\rangle + e^{i\theta} |101\rangle}{\sqrt{2}} ,\qquad(7)$$

$$|\psi_{\mathrm{B},3}\left(\theta\right)\rangle = \frac{|000\rangle + e^{i\theta}\left|011\rangle}{\sqrt{2}} \,. \tag{8}$$

The overlaid white × symbol in Fig. 2 represents the initial state  $|\psi_i\rangle$  used for generating the plots in Fig. 1. For this initial state  $|\psi_i\rangle$ , the final state  $|\psi_f\rangle$  is approximately  $|\psi_f\rangle \simeq (1/2) (|000\rangle + i |010\rangle + i |101\rangle - |111\rangle)$ . The values of the partial entanglements  $\tau_{23}$ ,  $\tau_{31}$ , and  $\tau_{12}$  at the final time  $t = 50/\gamma$  are shown in Fig. 2 (1), (2) and (3), respectively, as a function of  $\epsilon_1$  and  $\epsilon_2$ .

As is demonstrated by Fig. 1(c3) and Fig. 3(c3), generically,  $\tau_{12} \to 0$  in the limit  $t \to \infty$ , due to the partial disentanglement generated by the term proportional to  $Q_{12} - \langle Q_{12} \rangle$  in Eq. (1). The set  $V_n \subset H$  is a subset of the Hilbert space  $H = H_3 \otimes H_2 \otimes H_1$  containing all state vectors  $|\psi\rangle \in H$ , for which  $k_n = 1$ , where  $n \in \{1, 2, 3\}$ , i.e. the *n*'th spin is fully separable for all  $|\psi\rangle \in V_n$ . Note that  $V_1 \cap V_2 = V_2 \cap V_3 = V_3 \cap V_1$  is the set of fully disentangled states (i.e.  $k_1 = k_2 = k_3 = 1$ ), which is denoted by V. Let  $|\psi_{\infty}\rangle$  be the solution of Eq. (1) in the limit  $t \to \infty$ , for which  $\tau_{12} = 0$  (i.e. entanglement between spin 1 and spin 2 vanishes). Generically, either  $|\psi_{\infty}\rangle \subset V_1$  (for this case  $k_1 = 1$ ,  $k_2 = k_3 \leq 1$ ,



FIG. 4: Partial entanglements for  $|\psi_i\rangle \sim |\psi_{B,3}(\pi)\rangle + i\epsilon_1 |\psi_{B,1}(\pi)\rangle + i\epsilon_2 |\psi_{B,2}(\pi)\rangle$ . The values of  $\tau_{23}$ ,  $\tau_{31}$ , and  $\tau_{12}$  at the final time  $t = 50/\gamma$  are shown in (1), (2) and (3), respectively, as a function of  $\epsilon_1$  and  $\epsilon_2$ . The overlaid white  $\times$  symbol represents the initial state  $|\psi_i\rangle$  used for generating the plots in Fig. 3.

 $\tau_{12} = \tau_{31} = 0$ , and  $\tau_{23} \ge 0$ ), or  $|\psi_{\infty}\rangle \subset V_2$  (for this case  $k_2 = 1, k_3 = k_1 \le 1, \tau_{12} = \tau_{23} = 0$ , and  $\tau_{31} \ge 0$ ). Note that  $|\psi_{\infty}\rangle \subset V_2$  for both examples shown in Fig. 1 and in Fig. 3. For the general case, the entire Hilbert space H is divided into two basins of attraction,  $B_1$  and  $B_2$ . The basin  $B_n$  is the set of all initial states  $|\psi_1\rangle \in H$ , for which in the limit  $t \to \infty$  the final state  $|\psi_{\infty}\rangle \subset V_n$ , i.e.  $k_n = 1$ , where  $n \in \{1, 2\}$ .

As can be seen from Fig. 2, the GHZ state vector  $|\psi_{\text{GHZ}}\rangle$  lies on the separatrix between the basins of attraction  $B_1$  and  $B_2$ . The strong dependency of  $|\psi_{\infty}\rangle$  on the initial state  $|\psi_i\rangle$ , which becomes extreme in the vicinity of  $|\psi_{\text{GHZ}}\rangle$ , resembles the butterfly effect. As is demonstrated below in Fig. 4, a similar butterfly effect occurs near the state vector  $|\psi_{\text{B},3}(\pi)\rangle$ .

For the plots shown in Fig. 3 and Fig. 4 , the initial state vector  $|\psi_i\rangle$  is given by  $|\psi_i\rangle \sim |\psi_{B,3}(\pi)\rangle + i\epsilon_1 |\psi_{B,1}(\pi)\rangle + i\epsilon_2 |\psi_{B,2}(\pi)\rangle$ . For the example shown in Fig. 3,  $\epsilon_1 = 0$  and  $\epsilon_2 = 9 \times 10^{-5}$ . This initial state is indicated by the white × symbol in Fig. 4. The corresponding final state  $|\psi_f\rangle$  is approximately  $|\psi_f\rangle \simeq |\psi_{B,2}(-\pi/2)\rangle$ . As can be seen from Fig. 4, the butterfly effect occurs near the state  $|\psi_{B,3}(\pi)\rangle$ , which also (as the state  $|\psi_{GHZ}\rangle$ ) lies on the separatrix between the basins of attraction  $B_1$  and  $B_2$ .

Summary - The proposed modified Schrödinger equation (1) demonstrates a way to extend quantum mechanics to enable processes of deterministic partial disentanglement in a multipartite system. Conflicts with both principles of causality and separability can be avoided by postulating that partial disentanglement between two given subsystems is active only during the time when they interact [29]. This assumption of *local disentanglement* implies that the disentanglement rate  $\gamma$  in Eq. (1) vanishes during the time when subsystems are decoupled. Further theoretical study is needed to determine whether quantum mechanics can be self-consistently reformulated based on deterministic dynamics. Alternative theoretical models for the process of quantum measurement can be experimentally tested [43].

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## Appendix A: The function $\tau\left(\left|\psi\right\rangle\right)$

For a given Hermitian operator  $\mathcal{Q} = \mathcal{Q}^{\dagger}$ , the function  $\tau (|\psi\rangle)$ , which maps a ket vector  $|\psi\rangle$  to a real number  $\tau$ , is defined by

$$\tau\left(\left|\psi\right\rangle\right) \equiv \frac{\left\langle\psi\right|\mathcal{Q}\left|\psi\right\rangle}{\left\langle\psi\right|\psi\right\rangle} \,. \tag{A1}$$

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Let  $\Delta_{\tau} = \tau \left( |\psi\rangle + \epsilon |\delta\rangle \right) - \tau \left( |\psi\rangle \right)$ , where  $\epsilon$  is real, and  $|\delta\rangle$ is a ket vector. For  $\langle \psi |\psi\rangle = 1$  the following holds  $\Delta_{\tau} = 2\epsilon \operatorname{Re} \langle \delta | q_{\psi} \rangle + O(\epsilon^2)$ , where  $|q_{\psi}\rangle = (\mathcal{Q} - \langle \psi | \mathcal{Q} |\psi\rangle) |\psi\rangle$ . Within the set of normalized ket vectors, the term  $\operatorname{Re} \langle \delta | q_{\psi} \rangle$  is maximized for  $|\delta\rangle = |q_{\psi}\rangle / \sqrt{\langle q_{\psi} | q_{\psi} \rangle}$ . Moreover,  $\langle \psi | q_{\psi} \rangle = 0$  provided that  $\langle \psi | \psi \rangle = 1$ . Based on these observations, the nonlinear term added to the Schrödinger equation (1) is chosen to be proportional to  $|q_{\psi}\rangle$ .

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