The spontaneous disentanglement hypothesis and causality

Eyal Buks*
Andrew and Erna Viterbi Department of Electrical Engineering, Technion, Haifa 32000, Israel
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The hypothesis that disentanglement spontaneously occurs in quantum systems is motivated by some outstanding issues in the foundations of quantum mechanics. However, for some cases, spontaneous disentanglement enables the violation of the causality principle. To mitigate the conflict with causality, a formulation for the hypothesis, which is based on the maximum entropy principle, is proposed. The method of Lagrange multipliers is implemented to ensure consistency with causality. The proposed formulation is applicable for any quantum system having a Hilbert space of finite dimensionality.

Introduction – Unitary time evolution of a quantum state vector is governed by the Schrödinger equation. Standard quantum mechanics (QM) is commonly formulated based on the assumption that the unitary time evolution is supplemented by two additional auxiliary processes, which are both not derivable from the Schrödinger equation. The first one is a collapse of the state vector, which occurs when a measurement is performed, and the second one is thermalization. As was first shown by Schrödinger [1], the process of collapse gives rise to an internal inconsistency [2], which became known as the problem of quantum measurement. The conflict between the time–reversibility of the Schrödinger equation and the time–irreversibility of the process of thermalization is commonly referred to as the arrow of time problem.

Unitary time evolution in standard QM is governed by linear equations of motion. Both the measurement and the arrow of time problems have motivated the study of a variety of nonlinear extensions to QM [3–14]. Some of the proposed nonlinear extensions yield a spontaneous collapse of the state vector [15–20]. The collapse gives rise to disentanglement between a quantum system and its measuring apparatus. The generation of disentanglement, however, requires nonlinearity, because the subset of fully disentangled states within the Hilbert space of a given multipartite quantum system is generally not a linear subspace, and consequently, disentanglement cannot be generated by equations of motion that obey the superposition principle. Moreover, thermalization, which can be described as an entropy maximization process [21], requires nonlinearity, because the entropy $\sigma = \langle -\log \rho \rangle$ is a nonlinear function of the density operator ρ [4, 22]. Note that multi–stabilities in finite systems, which are excluded by standard QM [23], become possible provided that nonlinearity is permitted [24].

For some cases, however, nonlinear quantum dynamics may give rise to conflicts with well-established physical principles, such as separability [25–27] and causality [28–35]. The conflict with Einstein's causality principle is commonly demonstrated by showing that nonlinear dynamics enable superluminal (i.e. faster than light) signaling.

Note that, even in standard QM, where unitary time evolution is linear, the problem of superluminal signaling cannot be fully avoided (e.g. see [36, 37]). The vacuum speed of light c does not appear in the Schrödinger equation, and consequently, a speed limit related to c cannot be derived from standard, and non–relativistic, QM. The relativistic version of QM partially addresses some of these difficulties. However, it has been shown that the Dirac equation, similarly to the Schrödinger one, can give rise to superluminal tunneling (e.g. see [38]). While standard QM does not exclude superluminal signaling, one may argue that, for some cases, the realization of such signaling, which is forbidden by the principle of causality, is practically difficult to implement. On the other hand, in his seminal paper [39], Gisin has shown that in the presence of quantum entanglement, the generation of superluminal signaling becomes relatively simple, provided that quantum time evolution is nonlinear.

An example protocol to realize the Gisin's superluminal telegraph is described below. Consider a system composed of two subsystems, labeled as A and B, respectively [40]. It is assumed that nonlinearity stems from the process of spontaneous disentanglement [41]. Alice owns subsystem A, which is a *single* qubit, whereas Bob owns subsystem B, which is a *pair* containing two qubits. The state of the entire system $|\psi\rangle$ is assumed to be a Greenberger Horne Zeilinger (GHZ) state given by (subscripts A and B refer to Alice and Bob, respectively)

$$\begin{aligned} |\psi\rangle &= \frac{|\uparrow\rangle_{\mathrm{A}} \otimes |\uparrow\uparrow\rangle_{\mathrm{B}} - |\downarrow\rangle_{\mathrm{A}} \otimes |\downarrow\downarrow\rangle_{\mathrm{B}}}{\sqrt{2}} \\ &= \frac{|\longrightarrow\rangle_{\mathrm{A}} \otimes |-\rangle_{\mathrm{B}} + |\longleftarrow\rangle_{\mathrm{A}} \otimes |+\rangle_{\mathrm{B}}}{\sqrt{2}} \ . \end{aligned} \tag{1}$$

The arrow symbols $\uparrow$, $\downarrow$, $\longrightarrow$ and $\longleftarrow$ label eigenvectors of the matrix $\boldsymbol{\sigma} \cdot \hat{\mathbf{u}}$, with an eigenvalue of $+1$, where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli spin matrix vector, and the unit vector $\hat{\mathbf{u}}$ is pointing in the $\hat{\mathbf{z}}$, $-\hat{\mathbf{z}}$, $\hat{\mathbf{x}}$ and $-\hat{\mathbf{x}}$, directions, respectively. Since both $|\uparrow\uparrow\rangle_{\mathrm{B}}$ and $|\downarrow\downarrow\rangle_{\mathrm{B}}$ are product states, disentanglement *within* subsystem B has no effect (disentanglement between subsystems A and B is disregarded, for simplicity). However, if Alice performs a quantum measurement in the basis

*Electronic address: eyal@ee.technion.ac.il

$\{|\longrightarrow\rangle_{\mathrm{A}}, |\longleftarrow\rangle_{\mathrm{A}}\}$, where $|\longrightarrow\rangle_{\mathrm{A}} = (|\uparrow\rangle_{\mathrm{A}} + |\downarrow\rangle_{\mathrm{A}})/\sqrt{2}$ and $|\longleftarrow\rangle_{\mathrm{A}} = (|\uparrow\rangle_{\mathrm{A}} - |\downarrow\rangle_{\mathrm{A}})/\sqrt{2}$, the state of subsystem B is expected to collapse into the state $|+\rangle_{\mathrm{B}}$, with probability 1/2, or into the state $|-\rangle_{\mathrm{B}}$,with the same probability of 1/2, where $|\pm\rangle_{\mathrm{B}} = (|\uparrow\uparrow\rangle_{\mathrm{B}} \pm |\downarrow\downarrow\rangle_{\mathrm{B}})/\sqrt{2}$. Both states $|+\rangle_{\mathrm{B}}$ and $|-\rangle_{\mathrm{B}}$ are fully entangled, and thus a disentanglement process occurring after Alice's measurement, is expected to give rise to an impact that can be detected by Bob (for example, by measuring the expectation values of Bell operators for the pair he owns). This detectability arguably enables superluminal signaling.

For the same above–discussed protocol, predictions that are derived from the spontaneous disentanglement hypothesis are entirely different. For example, according to this hypothesis, rather than performing measurements, all Alice and Bob can do is turn on and off coupling between the quantum system they own and measuring apparatuses. Moreover, no collapse occurs within the framework of this hypothesis. Nevertheless, for some cases, disentanglement of a spatially extended quantum system may give rise to a conflict with the causality principle. In the current study, this conflict is explored, and a way to avoid it is proposed.

The collapse postulate – Unitary time evolution is linear according to standard QM. This linearity is one of the assumptions that are commonly used to show that superluminal signaling is excluded. However, as is discussed below, another important assumption is related to the way the collapse postulate is formulated. Consider a bipartite system, composed of a quantum subsystem, which is labeled by the letter a, and a measuring apparatus, which is labeled by the letter b. The subsystems' density operators ρ_{a} and ρ_{b} are derived from the density operator ρ of the composed bipartite system by partial tracing, i.e. $\rho_{\mathrm{a}} = \mathrm{Tr}_{\mathrm{b}}\,\rho$ and $\rho_{\mathrm{b}} = \mathrm{Tr}_{\mathrm{a}}\,\rho$.

In standard QM, a measurement is described as a process having two steps. In the first one, a pure entangled state $|\psi\rangle$ is generated by the unitary time evolution that is induced by the coupling between the quantum system and its measuring apparatus. The Schmidt decomposition can be applied to express the pure state $|\psi\rangle$ as

$$|\psi\rangle = \sum_s p_s^{1/2} |s\rangle_{\mathrm{a}} \otimes |s\rangle_{\mathrm{b}} \ , \tag{2}$$

where $0 \leq p_s \leq 1$, $\sum_s p_s = 1$, and where the set $\{|s\rangle_{\mathrm{a}}\}$ ($\{|s\rangle_{\mathrm{b}}\}$) forms an orthonormal basis spanning the Hilbert space of subsystem a (b). In the Schmidt basis, the reduced density operators are given by $\rho_{\mathrm{a}} = \sum_s p_s |s\rangle_{\mathrm{a}\,\mathrm{a}}\langle s|$ and $\rho_{\mathrm{b}} = \sum_s p_s |s\rangle_{\mathrm{b}\,\mathrm{b}}\langle s|$. According to the collapse postulate, under some unspecified conditions, the density operator, which prior to the collapse is given by $\rho = |\psi\rangle\langle\psi|$, undergoes an abrupt change. When one of the subsystems, or both, are spatially extended, this change generally may give rise to a conflict with the causality principle. However, such a conflict can be avoided, provided that the collapse is postulated to give rise to a Nakajima–Zwanzig projection [42, 43], for which ρ is mapped into the state $\rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}$, where the tensor product $\rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}$ is given by [see Eq. (8.349) of [44]]

$$\rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}} = \sum_{s',s''} p_{s'} p_{s''} |s', s''\rangle \langle s', s''| \ , \tag{3}$$

and where $|s', s''\rangle = |s'\rangle_{\mathrm{a}} \otimes |s''\rangle_{\mathrm{b}}$.

Note that the assumption that the collapse gives rise a Nakajima–Zwanzig projection (i.e. $\rho \rightarrow \rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}$) [45], implies the Born rule [46–48]. Moreover, the same assumption implies stochasticity, and excludes deterministic time evolution, since a Nakajima–Zwanzig projection generally maps pure states into mixed ones. For the Nakajima–Zwanzig projection process $\rho \rightarrow \rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}$, both reduced density operators ρ_{a} and ρ_{b} are unchanged, and consequently superluminal signaling is excluded, since the collapse has no impact on any subsystem's property. On the other hand, for some alternative formulations of the collapse postulate, superluminal signaling can become possible. This observation suggests that both stochasticity and the Born rule of standard QM can be partially attributed to the causality principle, and to the requirement that superluminal signaling must be excluded [49–52].

The spontaneous disentanglement hypothesis is formulated using nonlinear equations of motion, which are described in the next section. In the following section, a method to mitigate the conflict with the causality principle, which is based on the same assumption, that the reduced density operators ρ_{a} and ρ_{b} are kept fixed, is proposed.

Nonlinear extension – Consider the case where, to first order in the time interval τ, the density operator ρ evolves according to [8, 10, 53–55]

$$\rho(t+\tau) = \sum_{k\in\{0,1\}} K_k \rho(t) K_k^{\dagger} + O\left(\tau^2\right) \ , \tag{4}$$

where $K_0 = 1 - \left(i\hbar^{-1}\mathcal{H} + \Theta\right)\tau$ and $K_1 = \sqrt{2\langle\Theta\rangle\tau}$ are Kraus operators, which satisfy the norm conservation condition $\left\langle K_0^{\dagger}K_0 + K_1^{\dagger}K_1 \right\rangle = 1 + O\left(\tau^2\right)$ [56], $\hbar$ is the reduced Planck's constant, $\mathcal{H} = \mathcal{H}^{\dagger}$ is the system's Hamiltonian, the positive semi–definite operator Θ is allowed to depend on ρ, and $\langle\Theta\rangle = \mathrm{Tr}(\Theta\rho)$. Alternatively, the time evolution of ρ can be described using a master equation given by [see Eq. (4)]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = i\hbar^{-1}[\rho, \mathcal{H}] + \Omega(\Theta) \ , \tag{5}$$

where for a general operator X, the operator $\Omega(X)$ is given by $\Omega(X) = -X\rho - \rho X + 2\langle X\rangle\rho$.

The master equation (5) is equivalent to a stochastic Langevin–Schrödinger equation for the state vector $|\psi\rangle$ given by [57, 58]

$$\frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = \left(-i\hbar^{-1}\mathcal{H} + \sqrt{2\langle\Theta\rangle}\xi(t) - \Theta\right)|\psi\rangle \ , \tag{6}$$

where $\langle\Theta\rangle = \langle\psi|\,\Theta\,|\psi\rangle$. The white noise term $\xi(t)$ has a vanishing averaged value, i.e. $\overline{\xi(t)} = 0$, and a correlation function given by $\overline{\xi(t')\,\xi^*(t'')} = \delta(t'-t'')$, where overbar denotes time averaging. Norm is conserved by both the modified Schrödinger equation (6) and the modified master equation (5) [note that Eq. (6) yields $\overline{(\mathrm{d}/\mathrm{d}t)\,\langle\psi\,|\psi\rangle} = 0$, provided that initially $\langle\psi\,|\psi\rangle = 1$, and Eq. (5) yields $(\mathrm{d}/\mathrm{d}t)\,\mathrm{Tr}\,\rho = 0$, provided that initially $\mathrm{Tr}\,\rho = 1$]. Moreover, positivity [59] of the density matrix ρ is conserved by the modified master equation (5) [see Eq. (2.202) of Ref. [44]].

A Langevin–Schrödinger equation having the form given by Eq. (6) can be derived from a modified master equation having a given $\Omega(\Theta)$ term [see Eq.(5)], provided that the mapping $\Omega(X) = -X\rho - \rho X + 2\langle X\rangle\rho$ can be inverted. Any bounded operator X can be decomposed as $X = X_0 + X_1$, where X_1 is traceless, and X_0, which is given by $X_0 = \left(d^{-1}\,\mathrm{Tr}\,X\right)I$, is proportional to the identity operator I, where d is the dimensionality (note that $\mathrm{Tr}\,X_0 = \mathrm{Tr}\,X$). The following holds $\Omega(X) = \Omega(X_1)$, and thus the mapping $\Omega(X)$ is generally not invertible [i.e. a given operator $\Omega(X)$ does not uniquely determine the operator X, since $\Omega(X)$ is independent of X_0]. To allow invertibility, the mapping $\Omega(X)$ is replaced by $\tilde{\Omega}(X)$, which is given by $\tilde{\Omega}(X) = \Omega(X) + I\,\mathrm{Tr}\,X$, and for which the following holds $\tilde{\Omega}(X) = \Omega(X_1) + I\,\mathrm{Tr}\,X_0$. The mapping $\tilde{\Omega}(X)$ is invertible if and only if $\dim(\ker\rho) = 0$, as can be shown by expressing ρ in terms of its normalized eigenvectors $|\phi_n\rangle$ and eigenvalues $r_n \in [0,1]$ as $\rho = \sum_n r_n\,|\phi_n\rangle\langle\phi_n|$, and by expressing X in the same basis as $X = \sum_{n,m} x_{n,m}\,|\phi_n\rangle\langle\phi_m|$, where $x_{n,m} = \langle\phi_n|\,X\,|\phi_m\rangle$. For the case where $\dim(\ker\rho) > 0$, i.e. $r_n = 0$ for some n, the following holds $\Omega(|\phi_n\rangle\langle\phi_n|) = 0$, thus $\tilde{\Omega}\left(|\phi_n\rangle\langle\phi_n| - d^{-1}I\right) = 0$, hence $\tilde{\Omega}(X)$ is not invertible. On the other hand, for the case where $\dim(\ker\rho) = 0$, i.e. $r_n > 0$ for all n, the mapping $\tilde{\Omega}(X)$ is invertible, since for this case the only solution for $\tilde{\Omega}(X) = 0$ is $X = 0$ [note that $\tilde{\Omega}(X) = 0$ implies that $\mathrm{Tr}\,X = 0$ and $(r_m + r_n)\,x_{n,m} = 2\langle X\rangle r_n\delta_{n,m}$].

For a general explicitly time–independent observable A, the modified master equation (5) yields

$$\frac{\mathrm{d}\langle A\rangle}{\mathrm{d}t} = -i\hbar^{-1}\langle[A,\mathcal{H}]\rangle - \langle\Delta_A\Delta_\Theta + \Delta_\Theta\Delta_A\rangle\ , \tag{7}$$

where $\Delta_A = A - \langle A\rangle$ and $\Delta_\Theta = \Theta - \langle\Theta\rangle$. Note that for the case where $\mathcal{H} = 0$ and $A = \Theta$, Eq. (7) yields$\langle\Theta\rangle/\mathrm{d}t = -2\left\langle\Delta_\Theta^2\right\rangle \leq 0$ for a fixed Θ. This result suggests that the nonlinear term in the modified master equation (5) gives rise to the suppression of the expectation value $\langle\Theta\rangle$. Hence, the nonlinear term can be employed to suppress a given physical property, provided that $\langle\Theta\rangle$ quantifies that property. As is discussed below, this suppression can be used to generate both thermalization and disentanglement.

Thermalization – According to Jaynes' principle [21, 60], the density operator of a given system in thermal equilibrium ρ_0 maximizes the entropy $\sigma = \langle-\log\rho\rangle$ under some constraints. Alternatively, ρ_0 can be derived by minimizing a free energy, which can be defined for any given set of linear constraints. For example, when the energy expectation value $\langle\mathcal{H}\rangle$ is constrained ($\mathcal{H}$, which is assumed to be time–independent, is the Hamiltonian), the state of thermal equilibrium ρ_0, which is given by $\rho_0 = e^{-\beta\mathcal{H}}/\,\mathrm{Tr}\left(e^{-\beta\mathcal{H}}\right)$ [see Eq. (8.108) of Ref. [44]], can be found by minimizing the the Helmholtz free energy $\langle\mathcal{U}_\mathrm{H}\rangle$, which is given by $\langle\mathcal{U}_\mathrm{H}\rangle = \langle\mathcal{H}\rangle - \beta^{-1}\sigma$. The Lagrange multiplier β is given by $\beta = 1/(k_\mathrm{B}T)$, where k_B is the Boltzmann's constant, and T is the temperature. Thus, for this case, thermalization can be generated by the nonlinear term in the modified master equation (5), provided that the operator Θ is taken to be proportional to the Helmholtz free energy operator $\mathcal{U}_\mathrm{H} = \mathcal{H} + \beta^{-1}\log\rho$ [41]. As is discussed below, disentanglement can be generated in a similar way.

Subadditivity – The entropies of the quantum system σ_a, the measurement apparatus σ_b, and the bipartite composed system σ are given by $\sigma_\mathrm{a} = -\,\mathrm{Tr}\,(\rho_\mathrm{a}\log\rho_\mathrm{a})$, $\sigma_\mathrm{b} = -\,\mathrm{Tr}\,(\rho_\mathrm{b}\log\rho_\mathrm{b})$ and $\sigma = -\,\mathrm{Tr}\,(\rho\log\rho)$, respectively. The relative entropy $\sigma(\rho\parallel\rho_\mathrm{a}\otimes\rho_\mathrm{b}) = \sigma_\mathrm{a} + \sigma_\mathrm{b} - \sigma$ quantifies the quantum mutual information. It has been shown that relative entropy can be used to formulate entanglement area laws [61], and both the second [62] and third [63] laws of thermodynamics. As was discussed above, the conflict with the causality principle can be avoided, provided that the reduced density operators ρ_a (the quantum system) and ρ_b (the measurement apparatus) are unaffected by the process of disentanglement.

According to the Klein subadditivity inequality [64, 65]

$$\sigma \leq \sigma_\mathrm{a} + \sigma_\mathrm{b}\ , \tag{8}$$

and $\sigma = \sigma_\mathrm{a} + \sigma_\mathrm{b}$ if and only if $\rho = \rho_\mathrm{a}\otimes\rho_\mathrm{b}$. Hence $\rho_\mathrm{a}\otimes\rho_\mathrm{b}$ maximizes the entropy σ, under the constraints that both ρ_a and ρ_b are fixed. This observation suggests that the process of collapse can be mimicked by the modified master equation (5), provided that the operator Θ is constructed such that the expectation value $\langle-\Theta\rangle$ quantifies the total entropy σ, and that the constraints that fix both ρ_a and ρ_b are enforced. These constraints can be described in terms of the generalized Bloch vectors, which are defined in the next section.

The Bloch matrix and vectors – Let D_a, D_b and $D_\mathrm{H} = D_\mathrm{a}D_\mathrm{b}$ be the Hilbert space dimensionality of subsystem a, subsystem b, and the composed system, respectively (it is assumed that D_a, D_b and D_H are all finite). The generalized Gell-Mann set $\{\lambda_l\}$, which spans the $\mathrm{SU}(D_\mathrm{H})$ Lie algebra, contains $D_\mathrm{H}^2 - 1$ square $D_\mathrm{H}\times D_\mathrm{H}$ Hermitian matrices. For the case $D_\mathrm{H} = 2$ ($D_\mathrm{H} = 3$), the $D_\mathrm{H}^2 - 1 = 3$ ($D_\mathrm{H}^2 - 1 = 8$) elements are called Pauli (Gell-Mann) matrices. The Generalized Gell-Mann matrices are traceless, i.e. $\mathrm{Tr}\,\lambda_l = 0$, and they satisfy the orthogonality relation $(1/2)\,\mathrm{Tr}\,(\lambda_{l'}\lambda_{l''}) = \delta_{l',l''}$.

The generalized Gell-Mann $D_\mathrm{L}\times D_\mathrm{L}$ matrices corresponding to subsystem L, where $\mathrm{L}\in\{\mathrm{a},\mathrm{b}\}$, are denoted by $\lambda_l^{(\mathrm{L})}$, where $l \in \left\{1, 2, \cdots, D_\mathrm{L}^2 - 1\right\}$. Consider the set of $D_{\rm H}^2-1$ matrices $G^{\rm (ab)}=\left\{\Gamma_a^{\rm (a)}\otimes\Gamma_b^{\rm (b)}\right\}-\left\{\Gamma_0^{\rm (a)}\otimes\Gamma_0^{\rm (b)}\right\}$, where $a\in\left\{0,1,2,\cdots,D_{\rm a}^2-1\right\}$ and $b\in\left\{0,1,2,\cdots,D_{\rm b}^2-1\right\}$. For subsystem L, where $\rm L\in\{a,b\}$, the matrix $\Gamma_0^{\rm (L)}$ is defined by $\Gamma_0^{\rm (L)}=\left(2^{1/4}/D_{\rm L}^{1/2}\right)I_{\rm L}$, where $I_{\rm L}$ is the $D_{\rm L}\times D_{\rm L}$ identity matrix, and for $l\in\left\{1,2,\cdots,D_{\rm L}^2-1\right\}$ the matrix $\Gamma_l^{\rm (L)}$ is defined by $\Gamma_l^{\rm (L)}=2^{-1/4}\lambda_l^{\rm (L)}$.

With the help of the Kronecker matrix product identities $\operatorname{Tr}(X_1\otimes X_2)=\operatorname{Tr}X_1\operatorname{Tr}X_2$ and $(X_1\otimes X_2)(X_3\otimes X_4)=(X_1X_3)\otimes(X_2X_4)$, one finds that the set $G^{\rm (ab)}$ shares two properties with the Gell-Mann set $\{\lambda_l\}$ of the $D_{\rm H}$-dimensional Hilbert space. The first one is tracelessness $\operatorname{Tr}G_{a,b}=0$ for any $G_{a,b}\equiv\Gamma_a^{\rm (a)}\otimes\Gamma_b^{\rm (b)}\in G^{\rm (ab)}$ [recall that $G_{0,0}\notin G^{\rm (ab)}$], and the second one is orthogonality

$$\frac{\operatorname{Tr}\left(G_{a',b'}G_{a'',b''}\right)}{2}=\delta_{a',a''}\delta_{b',b''}\;. \tag{9}$$

The $D_{\rm a}^2\times D_{\rm b}^2$ matrix B, where $B_{a,b}=\langle G_{a,b}\rangle$, is henceforth referred to as the Bloch matrix [66, 67]. The following holds $B_{0,0}=\sqrt{2/\left(D_{\rm a}D_{\rm b}\right)}$ (matrix elements' numbering starts from 0), and $\operatorname{Tr}\left(BB^\dagger\right)=\operatorname{Tr}\left(B^\dagger B\right)=2\operatorname{Tr}\rho^2$ [see Eq. (9)]. The single subsystem Bloch vectors $\mathbf{P}_{\rm a}=\left(B_{1,0},B_{2,0},\cdots,B_{D_{\rm a}^2-1,0}\right)$ and $\mathbf{P}_{\rm b}=\left(B_{0,1},B_{0,2},\cdots,B_{0,D_{\rm b}^2-1}\right)$ are extracted from the first column and first row, respectively, of the Bloch matrix B.

Constraints – Let $\rho_{\rm ME}$ be the density operator of the composed system, which maximizes the entropy $\sigma=-\operatorname{Tr}(\rho\log\rho)$, under the constraints that both $\rho_{\rm a}$ and $\rho_{\rm b}$ are fixed (i.e. the Bloch vectors $\mathbf{P}_{\rm a}$ of subsystem a, and $\mathbf{P}_{\rm b}$ of subsystem b are fixed). The maximum entropy density operator $\rho_{\rm ME}$ can be found using the Lagrange multipliers method, which yields $\rho\doteq Z^{-1}\exp\left(-\boldsymbol{\alpha}_{\rm a}\cdot\boldsymbol{\Lambda}_{\rm a}-\boldsymbol{\alpha}_{\rm b}\cdot\boldsymbol{\Lambda}_{\rm b}\right)$, where both vectors $\boldsymbol{\alpha}_{\rm a}$ and $\boldsymbol{\alpha}_{\rm b}$ are real, the partition function Z is given by $Z=\operatorname{Tr}\exp\left(-\boldsymbol{\alpha}_{\rm a}\cdot\boldsymbol{\Lambda}_{\rm a}-\boldsymbol{\alpha}_{\rm b}\cdot\boldsymbol{\Lambda}_{\rm b}\right)$, and the vectors $\boldsymbol{\Lambda}_{\rm a}$ and $\boldsymbol{\Lambda}_{\rm b}$ of $D_{\rm H}\times D_{\rm H}$ matrices are given by $\boldsymbol{\Lambda}_{\rm a}=\left(G_{1,0},G_{2,0},\cdots,G_{D_{\rm a}^2-1,0}\right)$ and $\boldsymbol{\Lambda}_{\rm b}=\left(G_{0,1},G_{0,2},\cdots,G_{0,D_{\rm b}^2-1}\right)$ [21, 60]. Note that $\rho_{\rm ME}$ is positive semi definite, and that $\rho_{\rm ME}=\rho_{\rm a}\otimes\rho_{\rm b}$ [see inequality (8)].

For the quantum equations of motions (5) and (6), the constraints [68] that both $\rho_{\rm a}$ and $\rho_{\rm b}$ are fixed (for the case $\mathcal{H}=0$) are enforced by applying the transformation

$$\Theta\rightarrow\Theta'=\Theta+\sum_{a=0}^{d_{\rm a}^2}\sum_{b=0}^{d_{\rm b}^2}\eta_{ab}G_{a,b}\;. \tag{10}$$

The Lagrange coefficients η_{ab} are determined from the requirement that both $\rho_{\rm a}$ and $\rho_{\rm b}$ are fixed, which is expressed as a set of $D_{\rm H}^2$ constraints given by $\operatorname{Tr}\left(\Omega\left(\Theta'\right)G_{a,b}\right)=\operatorname{Tr}\left(\Omega\left(\Theta\right)G_{a,b}\right)$, for $(a=b=0)\vee(a\neq0\wedge b\neq0)$, and $\operatorname{Tr}\left(\Omega\left(\Theta'\right)G_{a,b}\right)=0$, for $(a=0\wedge b\neq0)\vee(a\neq0\wedge b=0)$ [the symbol $\wedge$ denotes logical and, the symbol $\vee$ denotes logical or, and see Eq. (5)]. Note that the Lagrange coefficients η_{ab} can be extracted from the constraints, provided that the mapping $\tilde{\Omega}(X)$ is invertible.

Two spin 1/2 system – The plots shown in Fig. 1 and in Fig. 2 demonstrate the disentanglement process for the case of a system composed of two spin 1/2 particles. For Fig. 1, the Hamiltonian vanishes (i.e. $\mathcal{H}=0$), and the initial state, which is pure, is given by $|\psi\rangle\langle\psi|$. In the Schmidt basis, the initial state vector $|\psi\rangle$ is expressed as $|\psi\rangle=\sqrt{p}\,|\uparrow\uparrow\rangle+\sqrt{1-p}\,|\downarrow\downarrow\rangle$ [see Eq. (2)], where $p\in[0,1]$ ($p=2/5$ for the plots shown in Fig. 1). The state's time evolution if found by numerically integrating the modified master equation (5). The nonlinear term is constructed using the operator $\Theta=\gamma\log\rho$, where γ is the rate of disentanglement. The plots on the left side of Fig. 1 display the time dependency of (a) the purity $\operatorname{Tr}\rho^2$, (b) the total entropy $\sigma=\langle-\log\rho\rangle$ and (c) the relative entropy [65, 69, 70] $\sigma\left(\rho\parallel\rho_{\rm a}\otimes\rho_{\rm b}\right)=\sigma_{\rm a}+\sigma_{\rm b}-\sigma$ [see inequality (8), and note that generally $\sigma\left(\rho'\parallel\rho''\right)\equiv\operatorname{Tr}\left(\rho'\left(\log\rho'-\log\rho''\right)\right)\geq0$].

For the plots that are black colored, no constraints are applied, whereas the color blue is used to label the plots that are obtained by applying the constraints [see Eq. (10)]. The 4×4 block of plots that are labeled by the capital letter B display the time dependency of the $4\times4=16$ entries $B_{a,b}$ of the Bloch matrix B. Note that, for the blue–colored plots (for which the constraints are applied), the single spin Bloch vectors $\mathbf{P}_{\rm a}=\left(B_{1,0},B_{2,0},B_{3,0}\right)$ and $\mathbf{P}_{\rm b}=\left(B_{0,1},B_{0,2},B_{0,3}\right)$ are unaffected by the disentanglement process. Nevertheless, as can be seen from Fig. 1(c), the efficiency of disentanglement is nearly unaffected by the constraints.

The effect of dipolar coupling is demonstrated by the plots shown in Fig. 2, which depict the time evolution of the single spin normalized Bloch vectors $\mathbf{k}_{\rm a}\equiv2^{1/2}\mathbf{P}_{\rm a}$ and $\mathbf{k}_{\rm b}\equiv2^{1/2}\mathbf{P}_{\rm b}$. The initial state $|\psi\rangle$ at time $t=0$, which is assumed to be both pure and fully disentangled, is labeled by a green $\times$ symbol, whereas a red $\times$ symbol is used to label the final state. The dipolar coupling is described by the Hamiltonian $\mathcal{H}$, which is given by $\mathcal{H}=\omega\sigma_3\otimes\sigma_3$, where ω denotes the dipolar coupling coefficient. Without applying the constraints, the system evolves to the fully mixed state, for which $\mathbf{k}_{\rm a}=\mathbf{k}_{\rm b}=0$ (see the red $\times$ symbols for the black colored plots). On the other hand, finite normalized Bloch vectors $\mathbf{k}_{\rm a}$ and $\mathbf{k}_{\rm b}$ are obtained in the long time limit when the constraints are applied (see the red $\times$ symbols for the blue colored plots). Note that in the long time limit both $\mathbf{k}_{\rm a}$ and $\mathbf{k}_{\rm b}$ are parallel (or anti–parallel) to the dipolar coupling direction (the z axis). Thus the time evolution generated by both the dipolar coupling $\sigma_3\otimes\sigma_3$ and the constrained process of disentanglement mimics a measurement of the spins' angular momentum z component (i.e. $k_{\rm a3}$ and $k_{\rm b3}$).

Discussion – The examples presented in Figs. 1 and 2

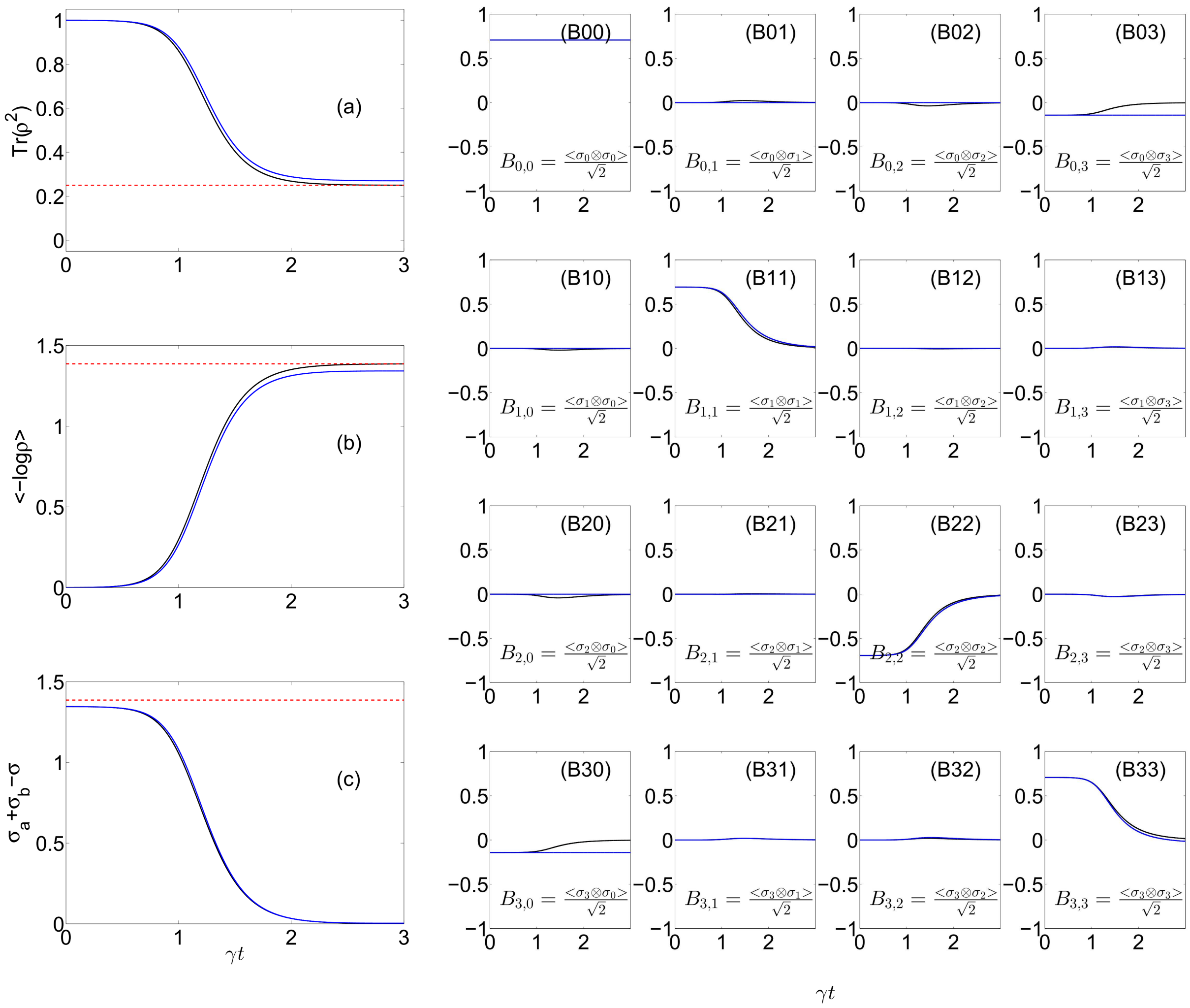

FIG. 1: Two spin 1/2 system. Plots generated without (with) applying the constraints are black (blue) colored. The (a) purity $\operatorname{Tr}\rho^2$, (b) total entropy $\sigma = \langle -\log\rho \rangle$ and (c) relative entropy $\sigma(\rho \parallel \rho_a \otimes \rho_b) = \sigma_a + \sigma_b - \sigma$ [see Eq. (8.162) of Ref. [44]] are plotted as a function of normalized time γt. The overlaid horizontal dotted red lines in (a), (b) and (c) represents the values of $1/4$, $\log 4$ and $\log 4$, respectively. The time dependency of the Bloch matrix element $B_{a,b}$ is shown in (Bab), where $a \in \{0,1,2,3\}$ and $b \in \{0,1,2,3\}$. The 2×2 identity matrix is denoted by σ_0, and σ_1, σ_2 and σ_3 are Pauli matrices.

demonstrate that the conflict with the principle of causality can be mitigated. However, the proposed formulation is based on non–relativistic QM, and consequently, full reconciliation with causality is seemingly unachievable within this framework (e.g. similarly to standard QM, superluminal tunneling cannot be excluded). Nevertheless, the proposed formulation allows incorporating unitary time evolution with the processes of disentanglement and thermalization, and it can be used to derive an effective model for some nonlinear effects in quantum systems.

Spontaneous disentanglement [41] makes the collapse postulate of QM redundant. Disentanglement has no effect on any product (i.e. disentangled) state, thus, all predictions of standard QM are unchanged in the absence of entanglement. For a multipartite system, disentanglement between any pair of subsystems can be introduced. Disentanglement is invariant under any subsystem unitary transformation, and it is applicable for both distinguishable and indistinguishable particles [71]. The spontaneous disentanglement hypothesis is falsifiable – its predictions are distinguishable from what is obtained from standard QM. Recently, the hypothesis has been experimentally tested using a spin resonator [72]. Further study is needed to experimentally test the hypothesis for other physical systems.

Summary – In the current study, the conflict between the spontaneous disentanglement hypothesis and the causality principle is explored. A formulation of the hypothesis, which is based on the maximum entropy principle, is proposed, and it is found that the conflict with the causality principle can be mitigated by introducing

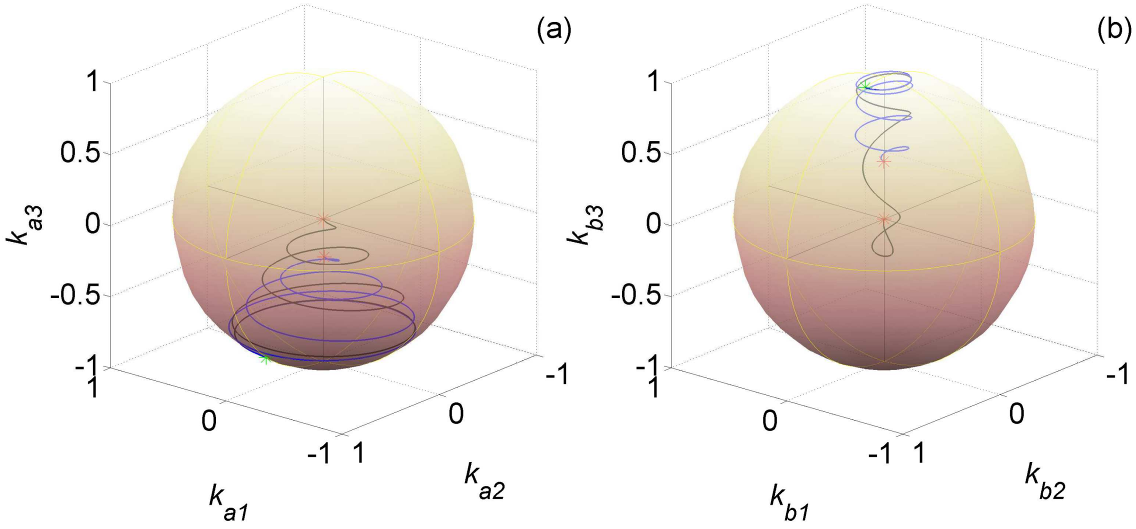

FIG. 2: Dipolar coupling. Time evolution of the single spin normalized Bloch vectors $\mathbf{k}_{\mathrm{a}}$ and $\mathbf{k}_{\mathrm{b}}$ is shown in (a) and (b), respectively. The color black (blue) is used to label the plots that have been obtained without (with) applying the constraints. The dipolar coupling coefficient is $\omega = 100$, and the rate of disentanglement is $\gamma = 3$. Note that initially $|\mathbf{k}_{\mathrm{a}}| = |\mathbf{k}_{\mathrm{b}}| = 1$ (see the points labeled by a green $\times$ symbol), since the initial state is both pure and fully disentangled.

constraints, which ensure that subsystems' properties are unaffected by the process of disentanglement.

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