Disentanglement in the macroscopic limit

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The recently proposed spontaneous disentanglement hypothesis is formulated using a modified Schrödinger equation having an added nonlinear term. The hypothesis is motivated by some outstanding issues in the foundations of quantum mechanics, including the problem of quantum measurement. Spontaneous disentanglement is explored in the current study for the macroscopic limit. This is done using some many–body models having known exact solutions. For the under–study models, it is found that non–local entanglement becomes unstable in the macroscopic limit. On the other hand, stability in the macroscopic limit of local entanglement is not excluded. These findings demonstrate that the spontaneous disentanglement hypothesis can bridge between the quantumness of the microscopic realm, and the classicalness of the macroscopic one.

Introduction – Unitary time evolution of a quantum state vector $|\psi\rangle$ is governed by the Schrödinger equation. It is commonly assumed that the unitary time evolution is supplemented by two auxiliary processes. One is thermalization, and the other is a collapse of the state vector [1, 2], which occurs when a measurement is performed. The collapse gives rise to disentanglement between a measured quantum system and its measuring apparatus [3]. The Schrödinger equation is linear in $|\psi\rangle$, whereas both auxiliary processes violate the principle of superposition, and consequently nonlinear equations of motion are needed to describe them [84]. Moreover, in contrast to the time–reversibility of the Schrödinger equation, both auxiliary processes are time–irreversible. The inability to describe both thermalization [4, 5] and disentanglement [6–11] processes using the Schrödinger equation has motivated the study of a variety of nonlinear extensions to quantum mechanics [12–23].

A nonlinear extension to quantum mechanics, which is based on the hypothesis that disentanglement spontaneously occurs in quantum systems, has been recently proposed [24]. Equations of motion, which are derived from this hypothesis, incorporate unitary time evolution with both auxiliary processes of thermalization and disentanglement. It was shown that this proposed nonlinear extension can account for some phenomena that are experimentally observed in quantum systems, including phase transitions, dynamical multi–stabilities and superconductivity [25–27]. These phenomena are commonly accounted for by employing the mean field approximation. However, the validity of this approximation is arguably questionable, in particular when it turns linear dynamics into nonlinear ones [28].

Quantum entanglement has been conclusively observed in all known microscopic physical systems (e.g. elementary particles, atoms and molecules), and in many mesoscopic systems. On the other hand, the applicability of standard quantum mechanics for macroscopic systems is arguably confutable [29–31]. In the current study, the proposed nonlinear equations of motion are analyzed for some many–body quantum models having known analytical solutions. In particular, the dependency on system size is studied, in order to explore whether the proposed nonlinear extension can describe the transition between quantumness in the microscopic limit, and classicalness in the macroscopic one. Our results demonstrate that under appropriate conditions, disentanglement can have a significant impact in the macroscopic limit, even when its effect in the microscopic limit is made arbitrarily small.

Spontaneous disentanglement hypothesis – A nonlinear extension to quantum dynamics can be formulated using nonlinear Kraus operators [16, 18, 32–34]. Consider the case where, to first order in the time interval τ, the density operator ρ evolves according to

$$\rho\left(t+\tau\right)=K_0\rho\left(t\right)K_0^{\dagger}+K_1\rho\left(t\right)K_1^{\dagger}+O\left(\tau^2\right)\ ,\qquad(1)$$

where $K_0=1-\left(i\hbar^{-1}\mathcal{H}+\Theta\right)\tau$ and $K_1=\sqrt{2\left\langle\Theta\right\rangle\tau}$ are Kraus operators, which satisfy the norm conservation condition $\left\langle K_0^{\dagger}K_0+K_1^{\dagger}K_1\right\rangle=1+O\left(\tau^2\right)$ [35], $\hbar$ is the reduced Planck's constant, $\mathcal{H}=\mathcal{H}^{\dagger}$ is the system's Hamiltonian, the positive semi–definite operator Θ is allowed to depend on ρ, and $\langle\Theta\rangle=\mathrm{Tr}\left(\Theta\rho\right)$. The corresponding time evolution of the system's state vector $|\psi\rangle$ can be described using a stochastic Langevin–Schrödinger equation [36, 37] [see Eq. (6) of Ref. [38]]. Alternatively, dynamics of the density operator ρ can be described using a positivity–preserving [39] master equation [see Eq. (5) of Ref. [38]].

For the case $\mathcal{H}=0$, and for a fixed operator Θ, Eq. (1) yields an equation of motion for $\langle\Theta\rangle$ given by $\mathrm{d}\left\langle\Theta\right\rangle/\mathrm{d}t=-2\left\langle\left(\Theta-\left\langle\Theta\right\rangle\right)^2\right\rangle$, and thus for this case the expectation value $\langle\Theta\rangle$ monotonically decreases with time. This observation suggests that the nonlinear term in Eq. (1) can be used to suppress a given physical property, provided that $\langle\Theta\rangle$ quantifies that property.

Two alternative methods to construct the operator Θ are employed in the current study. For the first one [24], which is briefly reviewed below in this section, the operator Θ is assumed to be given by $\Theta=\gamma_{\mathrm{H}}\mathcal{Q}_{\mathrm{H}}+\gamma_{\mathrm{D}}\mathcal{Q}_{\mathrm{D}}$, where both rates γ_{H} and γ_{D} are nonnegative real num-

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bers, and both operators $\mathcal{Q}_{\mathrm{H}}$ and $\mathcal{Q}_{\mathrm{D}}$ are Hermitian. The first term $\gamma_{\mathrm{H}}\mathcal{Q}_{\mathrm{H}}$ gives rise to thermalization [4, 40], whereas disentanglement is generated by the second term $\gamma_{\mathrm{D}}\mathcal{Q}_{\mathrm{D}}$. The second method is discussed in appendix A.

The construction of the operator $\mathcal{Q}_{\mathrm{H}}$, which is used for both methods to generate thermalization, is based on Jaynes' maximum entropy principle [41, 42]. The operator $\mathcal{Q}_{\mathrm{H}}$ is taken to be given by $\mathcal{Q}_{\mathrm{H}} = \beta\mathcal{U}_{\mathrm{H}}$, where $\mathcal{U}_{\mathrm{H}} = \mathcal{H} + \beta^{-1}\log\rho$ is the Helmholtz free energy operator, $\beta = 1/(k_{\mathrm{B}}T)$ is the thermal energy inverse, k_{B} is the Boltzmann's constant, and T is the temperature. For the case where $\mathcal{H}$ is time independent and $\gamma_{\mathrm{D}} = 0$ (i.e. no disentanglement), the thermal equilibrium density operator ρ_0, which is given by $\rho_0 = e^{-\beta\mathcal{H}}/\operatorname{Tr}\left(e^{-\beta\mathcal{H}}\right)$, is a steady state solution, for which the Helmholtz free energy $\langle\mathcal{U}_{\mathrm{H}}\rangle$ is minimized [4, 40, 41, 43].

The construction of the disentanglement operator $\mathcal{Q}_{\mathrm{D}}$ is explained in Ref. [24]. For a multipartite system that is composed of N subsystems it is given by

$$\mathcal{Q}_{\mathrm{D}} = \sum_{1\leq s'<s''\leq N} C_{s',s''} , \tag{2}$$

where the operator $C_{s',s''}$ is defined by

$$C_{s',s''} = \frac{1}{4}\sum_{n'=1}^{N_{s'}^2-1}\sum_{n''=1}^{N_{s''}^2-1}\frac{\Lambda_{n',n''}^{(s',s'')}\left\langle\Lambda_{n',n''}^{(s',s'')}\right\rangle}{1-(\min(N_{s'},N_{s''}))^{-2}} , \tag{3}$$

and where

$$\Lambda_{n',n''}^{(s',s'')} = \lambda_{n'}^{(s')}\otimes\lambda_{n''}^{(s'')} - \left\langle\lambda_{n'}^{(s')}\right\rangle\left\langle\lambda_{n''}^{(s'')}\right\rangle . \tag{4}$$

The generalized Gell-Mann matrices of subsystem s, where $s\in\{1,2,\cdots,N\}$, are denoted by $\lambda_n^{(s)}$, where $n\in\left\{1,2,\cdots,N_s^2-1\right\}$ and N_s is the Hilbert space dimensionality of subsystem s [see Eq. (8.397) of Ref. [44]]. The expectation value $\langle C_{s',s''}\rangle$, which is bounded by $\langle C_{s',s''}\rangle\in[0,1]$, quantifies the level of correlation between subsystems s' and s''. Note that $\langle C_{s',s''}\rangle$ is proportional to the *linear* relative entropy of entanglement [see Eq. (8.440) of Ref. [44]]. However, the linear relative entropy of entanglement can be used to approximate the relative entropy of entanglement only in a limited range [45].

Time evolution that is generated by Eq. (1) gives rise to the suppression of the expectation value $\langle\Theta\rangle = \gamma_{\mathrm{H}}\langle\mathcal{Q}_{\mathrm{H}}\rangle + \gamma_{\mathrm{D}}\langle\mathcal{Q}_{\mathrm{D}}\rangle$, which can be expressed as $\langle\Theta\rangle = \gamma_{\mathrm{H}}\beta\langle\mathcal{U}_{\mathrm{H,eff}}\rangle$, where $\mathcal{U}_{\mathrm{H,eff}} = \mathcal{U}_{\mathrm{H}} + \beta^{-1}(\gamma_{\mathrm{D}}/\gamma_{\mathrm{H}})\mathcal{Q}_{\mathrm{D}}$ is the effective Helmholtz free energy operator. The ratio $\gamma_{\mathrm{D}}/\gamma_{\mathrm{H}}$ determines the relative impact of disentanglement in comparison with thermalization.

Macroscopic limit – To explore the transition from the microscopic to the macroscopic realms, we study here the dependency of disentanglement's impact on system's size. This is done using some many–body models that describe physical systems composed of N subsystems (particles or sites) [46]. For a time independent Hamiltonian $\mathcal{H}$, The dimensionless parameter ζ is defined by $\zeta = \beta\operatorname{spr}(\mathcal{H})/q_{\mathrm{gs}}$, where $\operatorname{spr}(\mathcal{H})$ is the spread of $\mathcal{H}$ (i.e. the gap between largest and smallest energy eigenvalues), and q_{gs} is the ground state value of $\langle\mathcal{Q}_{\mathrm{D}}\rangle$. The impact of disentanglement in the macroscopic limit can be estimated by evaluating the dependency of ζ on system's size.

Commonly, many–body models become intractable in the macroscopic limit, i.e. for $N\gg 1$. However exact solutions have been found for some specific models. Here we focus on three such integrable many–body models: the Lieb-Mattis antiferromagnet [47], the Affleck-Kennedy-Lieb-Tasa (AKLT) ring [48, 49], and the Kitaev chain [50]. For both the Lieb-Mattis antiferromagnet and the AKLT ring, the system under study is composed of N distinguishable spin S particles. For the Lieb-Mattis model $S\in\{1/2,1,3/2,\cdots\}$, whereas $S=1$ for the AKLT model. Haldane has shown that the low energy properties of a one–dimensional spin system with an antiferromagnetic interaction depend on the parity of $2S$ [51, 52]. For odd $2S$ (i.e. for $S\in\{1/2,3/2,\cdots\}$), the ground state commonly becomes degenerate in the limit $N\to\infty$. On the other hand, for even $2S$ (i.e. for $S\in\{1,2,\cdots\}$) the energy gap between the ground state and the first excited state remains finite in the macroscopic limit of $N\to\infty$ (this gap is commonly called the Haldane energy gap) [53, 54]. Moreover, correlation functions exponentially decay for even $2S$. These properties are demonstrated by the exactly solvable AKLT model [48, 49]. The Kitaev model [50] allows exploring the macroscopic limit for a system made of indistinguishable (and spinless) particles. For all three under–study models, the known solutions allow analytically exploring the impact of disentanglement as a function of system's size.

Lieb-Mattis model – Consider a system composed of $2N$ distinguishable spin $1/2$ particles, where N is a positive integer. The angular momentum vector operator of the l'th spin is denoted by $\mathbf{S}_l$. The system is composed of two subsystems labelled as A and B, and containing N spins each. The Lieb–Mattis Hamiltonian $\mathcal{H}$ is given by [47]

$$\mathcal{H} = \frac{J}{N\hbar}\mathbf{S}_{\mathrm{A}}\cdot\mathbf{S}_{\mathrm{B}} , \tag{5}$$

where J is a real constant, $\mathbf{S}_{\mathrm{A}} = \sum_{l=1}^{N}\mathbf{S}_l$ and $\mathbf{S}_{\mathrm{B}} = \sum_{l=N+1}^{2N}\mathbf{S}_l$. To model an antiferromagnet (in the long–range limit), it is assumed that $J>0$.

For the classical (i.e. fully disentangled) Néel state $|N(\hat{\mathbf{n}})\rangle$, all spins belonging to the A (B) subsystem are pointing in the $\hat{\mathbf{n}}$ ($-\hat{\mathbf{n}}$) direction, where $\hat{\mathbf{n}}$ is a unit vector. The energy expectation value E_{N} of the Néel state is given by $E_{\mathrm{N}} = \langle N(\hat{\mathbf{n}})|\mathcal{H}|N(\hat{\mathbf{n}})\rangle = -N\hbar J/4$. The ground state $|\psi_{\mathrm{g}}\rangle$, which is entangled, has a lower energy E_{g} given by $E_{\mathrm{g}} = \langle\psi_{\mathrm{g}}|\mathcal{H}|\psi_{\mathrm{g}}\rangle = -(N\hbar J/4)(1+2/N)$

[55], and the state $|\psi_{\mathrm{g}}\rangle$ can be expressed as [56, 57]

$$|\psi_{\mathrm{g}}\rangle = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} \frac{(-1)^{\frac{N}{2}-m}}{\sqrt{N+1}} |m,-m\rangle \ , \tag{6}$$

where $|m,-m\rangle$ is a normalized common eigenvector of the operators $\mathbf{S}_{\mathrm{A}}^2$, $\mathbf{S}_{\mathrm{B}}^2$, $S_{\mathrm{A}z}$ and $S_{\mathrm{B}z}$, with eigenvalues $(N/2)(N/2+1)\hbar^2$, $(N/2)(N/2+1)\hbar^2$, $m\hbar$ and $-m\hbar$, respectively. Note that the ground state $|\psi_{\mathrm{g}}\rangle$ is not gaped (i.e. the energy separation between the ground and first excites states vanishes in the limit $N\to\infty$). This property enables a spontaneous symmetry breaking occurring in the macroscopic limit of $N\to\infty$ [58].

Consider a subsystem composed of two spins l' and l''. The two spins' reduced density operator $\rho_{(l',l'')}$ is evaluated by partial tracing of the total density operator ρ

$$\rho_{(l',l'')} = \mathrm{Tr}_{l\notin\{l',l''\}}\, \rho \ . \tag{7}$$

For the case where $l' \in \{1,2,\cdots,N\}$ and $l'' \in \{N+1,N+2,\cdots,2N\}$ (i.e. l' labels a spin in A, and l'' labels a spin in B), and for the ground state $|\psi_{\mathrm{g}}\rangle$, the matrix representation of $\rho_{(l',l'')}$ is given by [see Eq. (8.468) of [44]]

$$\rho_{(l',l'')} \doteq \begin{pmatrix} \frac{N-1}{6N} & 0 & 0 & 0 \\ 0 & \frac{2N+1}{6N} & -\frac{N+2}{6N} & 0 \\ 0 & -\frac{N+2}{6N} & \frac{2N+1}{6N} & 0 \\ 0 & 0 & 0 & \frac{N-1}{6N} \end{pmatrix} . \tag{8}$$

Note that $\rho_{(l',l'')}$ is independent on both l' and l'', $\mathrm{Tr}\,\rho_{(l',l'')} = 1$ and $\mathrm{Tr}\,\rho_{(l',l'')}^2 = \left(N^2+N+1\right)/\left(3N^2\right)$. The partial transpose of $\rho_{(l',l'')}$ has a negative eigenvalue given by $-1/(2N)$, thus, $\rho_{(l',l'')}$ is not separable according to the Peres–Horodecki criterion [59]. The two spin ground state entanglement $\tau_{\mathrm{TS}} \equiv \langle C_{l',l''}\rangle$ [see Eq. (3)] is given by

$$\tau_{\mathrm{TS}} = \frac{1}{9}\left(1+\frac{2}{N}\right)^2 . \tag{9}$$

Note that generally (i.e. for an arbitrary two spin 1/2 system), $\mathrm{Tr}\,\rho_{(l',l'')}^2$ is bounded by $\mathrm{Tr}\,\rho_{(l',l'')}^2 \in [1/4,1]$, and τ_{TS} is bounded by $\tau_{\mathrm{TS}} \in [0,1]$. For the ground state $|\psi_{\mathrm{g}}\rangle$, in the macroscopic limit, i.e. for $N\to\infty$, $\mathrm{Tr}\,\rho_{(l',l'')}^2 \to 1/3$ and $\tau_{\mathrm{TS}} \to 1/9$.

Entanglement [60] is evaluated below for the state $|\psi(p)\rangle$, which is given by

$$|\psi(p)\rangle = \frac{\sqrt{1-p}\,|\psi_{\mathrm{g}}\rangle + \sqrt{p}\,|N(\hat{\mathbf{z}})\rangle}{\left\|\sqrt{1-p}\,|\psi_{\mathrm{g}}\rangle + \sqrt{p}\,|N(\hat{\mathbf{z}})\rangle\right\|} , \tag{10}$$

where $p \in [0,1]$. The state $|\psi(p)\rangle$ represents a weighted average between the ground state $|\psi_{\mathrm{g}}\rangle$ and the Néel state $|N(\hat{\mathbf{z}})\rangle$ [note that $|\psi(p=0)\rangle = |\psi_{\mathrm{g}}\rangle$ and $|\psi(p=1)\rangle = |N(\hat{\mathbf{z}})\rangle$]. The plots in the top (first) row of

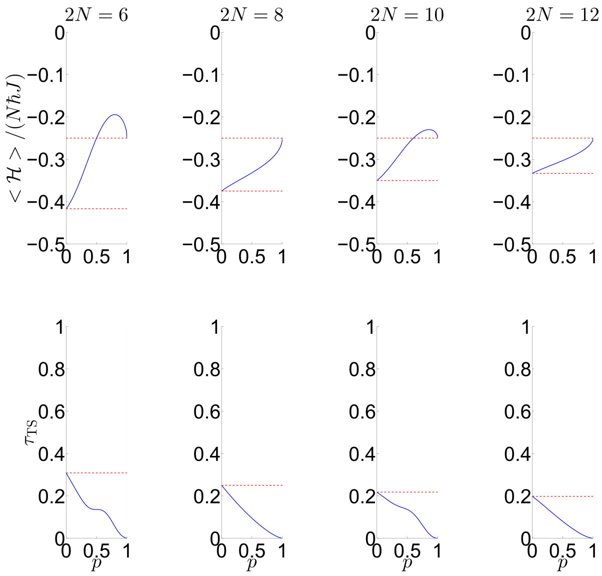

FIG. 1: The Lieb-Mattis model. The energy expectation value $\langle\mathcal{H}\rangle$ and the level of entanglement τ_{TS} are plotted as a function of p [see Eq. (10)] for $2N = 6$, 8, 10 and 12.

Fig. 1 display the normalized energy expectation value $\langle\psi(p)|\,\mathcal{H}\,|\psi(p)\rangle/(N\hbar J)$ as a function of p. The normalized energy expectation values of $-1/4$, for the Néel state, and of $-(1/4)(1+2/N)$, for the ground state, are represented by overlaid red dashed horizontal lines. The plots in the second row of Fig. 1 display the entanglement τ_{TS} between two spins l' and l'' (where $l' \in \{1,2,\cdots,N\}$ and $l'' \in \{N+1,N+2,\cdots,2N\}$) as a function of p. The overlaid red dashed horizontal lines in the second row of Fig. 1 represent the value given by Eq. (9) for the ground state entanglement τ_{TS}. The number N for the first (left), second, third and forth column in Fig. 1 is 3, 4, 5 and 6, respectively.

As was discussed above, the nonlinear term in the equations of motion suppresses the expectation value $\langle\Theta\rangle = \gamma_{\mathrm{H}}\langle\mathcal{Q}_{\mathrm{H}}\rangle + \gamma_{\mathrm{D}}\langle\mathcal{Q}_{\mathrm{D}}\rangle$. For the Lieb-Mattis model at low temperatures it is expected that the disentanglement term $\gamma_{\mathrm{D}}\langle\mathcal{Q}_{\mathrm{D}}\rangle$ becomes dominant in the macroscopic limit, because in this limit its ground state value is proportional to N^2 [see Eqs. (2) and (9)].

AKLT model – The impact of disentanglement on a gaped ground state can be explored using the AKLT model. Consider a one-dimensional array composed of L spin $S=1$ localized particles. The angular momentum vector operator of the l'th spin is denoted by $\mathbf{S}_l = (S_{l,x}, S_{l,y}, S_{l,z})$, where $l \in \{1,2,\cdots,L\}$. The Hilbert space of the single spin occupying site l, where $l \in \{1,2,\cdots,L\}$, is spanned by an orthonormal basis given by $\{|-;l\rangle, |0;l\rangle, |1;l\rangle\}$. The $q-$deformed matrix product state [61] is defined by $|\psi_q\rangle = \mathrm{Tr}(\mu_1\mu_2\cdots\mu_L)$,

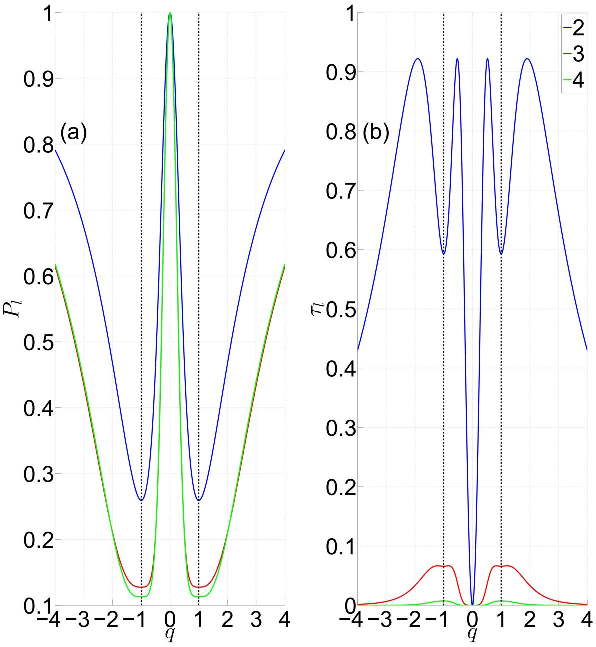

FIG. 2: AKLT model. Both (a) purity $P_l = \mathrm{Tr}\,\rho^2_{(1,l)}$ and (b) entanglement τ_l are plotted as a function of q. The colors blue, red and green are used for the site spacing l values of 2, 3 and 4, respectively. The overlaid black dashed vertical lines label the values $q = \pm 1$ (AKLT ground state).

where the matrices μ_l are given by [62]

$$\mu_l = \begin{pmatrix} q^{-1}\,|0;l\rangle & -\sqrt{q+q^{-1}}\,|+;l\rangle \\ \sqrt{q+q^{-1}}\,|-;l\rangle & -q\,|0;l\rangle \end{pmatrix} , \qquad (11)$$

and q is real (note that $|\psi_q\rangle$ is not normalized). The state $|\psi_q\rangle$ for the case $q = 1$ is the ground state of the AKLT Hamiltonian $\mathcal{H}_{\mathrm{AKLT}}$, which is given by

$$\mathcal{H}_{\mathrm{AKLT}} = \sum_{l=1}^{L} P_2^{(l,l+1)} , \qquad (12)$$

where $P_2^{(l,l+1)}$ is the projection operator corresponding to the pair of l and $l+1$ spins, and to the quantum number $S = 2$ [see Eq. (18.569) of [44]]. For the last term $l = L$, the projection operator $P_2^{(L,L+1)}$ represents the operator $P_2^{(L,1)}$. In the macroscopic limit, i.e. for $L \gg 1$, the Haldane energy gap [51, 52] Δ_{AKLT} for the AKLT Hamiltonian (12) is $\Delta_{\mathrm{AKLT}} \simeq 0.35$ [63, 64]. While the AKLT ground state $|\psi_{q=1}\rangle$ is entangled, for $q = 0$ the state $|\psi_q\rangle$ becomes fully disentangled.

The reduced density matrix corresponding to the subsystem composed of the first and the l'th sites is denoted by $\rho_{(1,l)}$. The plot in Fig. 2(a) shows the subsystem purity $\mathrm{Tr}\,\rho^2_{(1,l)}$ as a function of q. Note that $\mathrm{Tr}\,\rho^2_{(1,l)} = \mathrm{Tr}\left(\mathcal{M}_{\mathrm{W}}^{\dagger}\mathcal{M}_{\mathrm{W}}\right)$, where $\mathcal{M}_{\mathrm{W}}$ is the subsystem Weyl matrix [see Eq. (8.339) of Ref. [44]]. The entanglement [65, 66] between the first and the l'th sites, which is denoted by $\tau_l \equiv \langle C_{1.l}\rangle$ [see Eq. (3)], is plotted in Fig. 2(b) as a function of q. For the case $q = 1$, i.e. for the AKLT ground state, and in the limit $L \gg 1$, the following holds $\mathrm{Tr}\,\rho^2_{(1,l)} = 1/9 + (4/27)\,(1/9)^{l-2}$ and [see Eq. (18.1062)) of Ref. [44], and note that $l \geq 2$]

$$\tau_l = \frac{16}{27}\left(\frac{1}{9}\right)^{l-2} . \qquad (13)$$

Thus both $\mathrm{Tr}\,\rho^2_{(1,l)}$ and τ_l exponentially decrease as a function of site spacing l to their lowest allowed values (1/9 and 0, respectively). This observation suggests that entanglement in the AKLT ground state is local, and remote spins are uncorrelated.

Kitaev model – The effect of disentanglement on indistinguishable particles can be studied using the Kitaev model. Consider a one–dimensional array containing L sites that are occupied by *spinless* Fermions. The creation and annihilation operators corresponding to site $l \in \{1, 2, \cdots, L\}$ are denoted by $a_l^{\dagger}$ and a_l, respectively. The operators $a_l^{\dagger}$ and a_l satisfy Fermionic anti-commutation relations. The Kitaev chain Hamiltonian $\mathcal{H}_{\mathrm{K}}$ is given by

$$\mathcal{H}_{\mathrm{K}} = -t\sum_{l=1}^{L-1}\left(a_l^{\dagger}a_{l+1} + a_{l+1}^{\dagger}a_l - a_l^{\dagger}a_{l+1}^{\dagger} - a_{l+1}a_l\right) - \mu\sum_{l=1}^{L}\left(a_l^{\dagger}a_l - \frac{1}{2}\right) , \qquad (14)$$

where both t and μ are real constants. The Hamiltonian $\mathcal{H}_{\mathrm{K}}$ can be diagonalized using a Bogoliubov transformation. The plot in Fig.3(a) displays the normalized Bogoliubov eigenvalues η_l/t , where $l \in (1, 2, \cdots, 2L)$, as a function of the ratio μ/t. The two nearly vanishing eigenvalues in the region $\mu/t \lesssim 1$, which are red colored in Fig. 3(a), are commonly referred to as Majorana zero modes [67]. For the case $\mu = 0$, the ground state, which is doubly degenerate, has energy given by $-t\,(L-1)$.

The two sites l' and l'' reduced density operator is denoted by $\rho_{(l',l'')}$. For the case $\mu = 0$ the corresponding purity, which is given by $\mathrm{Tr}\,\rho^2_{(l',l'')} = 1/2$ [see Eq. (18.557) of Ref. [44]], is independent on both site indices l' and l''. Similarly, the level of entanglement between sites l' and l'' [68–73], which is denoted by $\tau_{(l',l'')} \equiv \langle C_{l',l''}\rangle$ [see Eq. (3)] is independent on both l' and l'' for the case $\mu = 0$. The dependency of the purity $\mathrm{Tr}\,\rho^2_{(1,L/2)}$ and entanglement $\tau_{(1,L/2)}$ on the ratio μ/t is shown in Fig. 3(b) and (c), respectively, for the case $L = 8$. For $\mu = 0$ and for arbitrary L

$$\tau_{(l',l'')} = \frac{1}{3} , \qquad (15)$$

and thus for this case the Kitaev ground state is non–locally entangled.

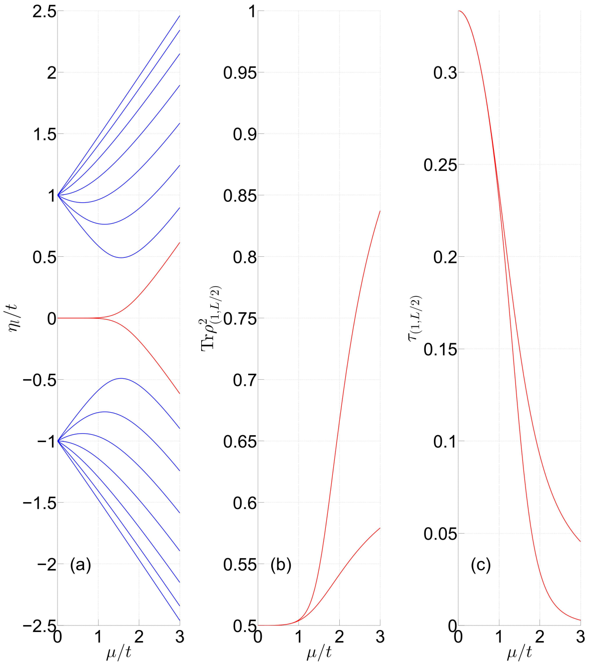

FIG. 3: Kitaev chain. The (a) Bogoliubov eigenvalues η_l/t, (b) two sites purity $\mathrm{Tr}\,\rho^2_{(1,L/2)}$ and (c) the two sites level of entanglement $\tau_{(1,L/2)}$ are plotted as as a function of the ratio μ/t, for the case $L = 8$.

Discussion – In the macroscopic limit (i.e. $N \gg 1$ for the Lieb-Mattis model, and $L \gg 1$ for the AKLT and Kitaev ones), the parameter q_{gs} (the ground state value of $\langle \mathcal{Q}_{\mathrm{D}} \rangle$) scales as N^2 for the case of the Lieb-Mattis model [see Eq. (9)], as L for the AKLT model [see Eq. (13)], and as L^2 for the Kitaev model with $\mu = 0$ [see Eq. (15)]. Thus, both the Lieb-Mattis and Kitaev (for $\mu = 0$) ground states become unstable in the macroscopic limit (the dimensionless parameter $\zeta \to 0$ in this limit for the Kitaev model). In contrast, the possibility that the AKLT ground state remains stable in the macroscopic limit is not excluded.

The expectation value of $\langle \mathcal{Q}_{\mathrm{D}} \rangle$ associated with the AKLT ground state obeys an area law [74, 75], and, consequently, correlation functions exponentially decay as a function of spacing between sites for this state [see Eq. (13)]. In contrast, long range correlations occur for both Lieb-Mattis and Kitaev (for $\mu = 0$) ground states. The examples presented in the current study, which are based on many–body models having known exact solutions, suggest a possible connection between the stability of macroscopic entangled states against the impact of disentanglement, and between the correlation length associated with the given state. However, further study is needed to incisively determine under what conditions a general macroscopic entangled state is kept stable against disentanglement.

Summary – The proposed nonlinear extension yields time evolution, which incorporates the processes of thermalization and collapse with unitary time evolution. In the current study, the transition from the microscopic limit to the macroscopic one is explored. Our findings demonstrate that, for some cases, the impact of disentanglement can be significant in the macroscopic limit, even when that impact is kept arbitrary small in the microscopic limit. In other words, for these cases, the spontaneous disentanglement hypothesis provides a possible way to reconcile between the quantumness of the microscopic realm, and the classicalness of the macroscopic one. The hypothesis is falsifiable, since it yields predictions that are distinguishable from what is obtained using alternative models. However, deriving predictions that are applicable in the macroscopic limit, and that are experimentally testable, is challenging, since commonly many–body models become intractable in the macroscopic limit (even without implementing disentanglement, which further complicates calculations).

Acknowledgments – Useful discussions with Nathan Argaman are acknowledged.

Appendix A: Disentanglement and causality

The nonlinear equation of motion (1) is constructed in the main text by assuming that the operator Θ is given by $\Theta = \gamma_{\mathrm{H}} \mathcal{Q}_{\mathrm{H}} + \gamma_{\mathrm{D}} \mathcal{Q}_{\mathrm{D}}$ (see Ref. [24]). For this construction method, however, and for a spatially extended quantum system, the resultant dynamics may give rise to an inconsistency with the Einstein's causality principle [76, 77]. Under some appropriate conditions, this inconsistency can be avoided by implementing an alternative method to construct the nonlinear equation of motion [38]. For this alternative method, stability of entangled quantum states in the macroscopic limit is discussed in this appendix.

In the first step of the alternative method, it is assumed that $\Theta = \gamma_{\mathrm{H}} \mathcal{Q}_{\mathrm{H}}$ (recall that $\mathcal{Q}_{\mathrm{H}} = \beta \mathcal{U}_{\mathrm{H}}$, where $\mathcal{U}_{\mathrm{H}}$ is the Helmholtz free energy operator). While the operator Θ is assumed to have only a thermalization term, disentanglement is introduced in the next step by enforcing an additional constraint [38].

Consider the case where the quantum system under study is composed of two subsystems, which are labeled by the letters a and b, respectively. The bipartite mutual information $\mathcal{I}_{\mathrm{a,b}}$ is expresses as the relative entropy $\sigma\left(\rho \parallel \rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}\right)$ between the density operators ρ and $\rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}$ [78]

$$\mathcal{I}_{\mathrm{a,b}} = \sigma_{\mathrm{a}} + \sigma_{\mathrm{b}} - \sigma = \sigma\left(\rho \parallel \rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}\right) , \tag{A1}$$

where the subsystems' entropies σ_{a} and σ_{b} are given by $\sigma_{\mathrm{a}} = -\mathrm{Tr}\left(\rho_{\mathrm{a}} \log \rho_{\mathrm{a}}\right)$ and $\sigma_{\mathrm{b}} = -\mathrm{Tr}\left(\rho_{\mathrm{b}} \log \rho_{\mathrm{b}}\right)$, respectively, and where the subsystems' reduced density operators ρ_{a} and ρ_{b} are derived from the density operator ρ of the composed system by partial tracing, i.e. $\rho_{\mathrm{a}} = \mathrm{Tr}_{\mathrm{b}}\, \rho$ and $\rho_{\mathrm{b}} = \mathrm{Tr}_{\mathrm{a}}\, \rho$. It has been shown that relative entropy and quantum mutual information can be used to formulate entanglement area laws [79], and both the second [80] and third [81] laws of thermodynamics. The Klein subadditivity inequality [78, 82], which states that $\sigma\left(\rho' \parallel \rho''\right) \equiv \operatorname{Tr}\left(\rho'\left(\log \rho' - \log \rho''\right)\right) \geq 0$, implies that $\sigma_{\mathrm{a}} + \sigma_{\mathrm{b}} - \sigma \geq 0$, and equality holds (i.e. $\sigma = \sigma_{\mathrm{a}} + \sigma_{\mathrm{b}}$) if and only if $\rho = \rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}$.

For a bipartite system, the total entropy σ can be expressed as $\sigma = \sigma_{\mathrm{a}} + \sigma_{\mathrm{b}} - \mathcal{I}_{\mathrm{a,b}}$. The subadditivity inequality implies that the disentangled state $\rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}$ maximizes the entropy σ, under the constraints that both subsystems' reduced density operators ρ_{a} and ρ_{b} are fixed. The mutual information minimum principle is implemented by enforcing these constraints. As is explained in Ref. [38], this can be done using the method of Lagrange multipliers. With these enforced constraints, the process of thermalization gives rise to the suppression of the quantum mutual information, which, in turn generates disentanglement between subsystems a and b. However, the constraints (that fix both ρ_{a} and ρ_{b}) ensure that this process of disentanglement has no impact on any single subsystem property. Hence, a conflict with causality, which can occur when the composed quantum system is spatially extended, is avoided.

Consider the case where the system's Hamiltonian $\mathcal{H}$ is time-independent, and can be expressed as $\mathcal{H} = \mathcal{H}_{\mathrm{a}} + \mathcal{H}_{\mathrm{b}} + V$, where $\mathcal{H}_{\mathrm{a}}$ and $\mathcal{H}_{\mathrm{b}}$ are Hamiltonians of subsystems a and b, respectively, and the term V represents interaction between the two subsystems. For this case, the entanglement area law [79] yields [recall that $\rho_0 = e^{-\beta\mathcal{H}} / \operatorname{Tr}\left(e^{-\beta\mathcal{H}}\right)$ is the thermal equilibrium density operator]

$$\sigma\left(\rho_0 \parallel \rho_{\mathrm{a}} \otimes \rho_{\mathrm{b}}\right) \leq 2\beta \left\|V\right\|_{\infty} , \tag{A2}$$

where the Schatten norm $\left\|V\right\|_{\infty}$ is defined by $\left\|V\right\|_{\infty} = \lim_{p\to\infty}\left(\operatorname{Tr}\left(|O|^p\right)\right)^{1/p}$ [see Eq. (17.232) of Ref. [44]].

The inequality (A2) can be used to estimate the stability of entangled states in the macroscopic limit. For the integrable models under study, the Schatten norm $\left\|V\right\|_{\infty}$ scales as N^2 for the case of the Lieb-Mattis model, as L for the AKLT model, and as L^2 for the Kitaev model with $\mu = 0$. Thus, for both the Lieb-Mattis and Kitaev (for $\mu = 0$) models, instability in the macroscopic limit cannot be excluded.

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